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Nominal Debt Dynamics, Credit Constraints and Monetary Policy

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Nominal Debt Dynamics, Credit Constraints and Monetary Policy*

Liam Graham and Stephen Wright

Abstract

We construct a dynamic general equilibrium model in which household debt is sticky in nominal terms and debtor households are credit constrained. Interest payments on debt contracts may be at floating rates or fixed for the duration of the contract. A key result is that a simple static Taylor Rule can result in a prolonged period in which real interest rates are cut rather than raised in response to an inflationary shock. We show how the proportion of fixed rate contracts affects the monetary transmission mechanism and its implications for the distributional effects of an inflationary shock.

KEYWORDS: nominal debt, dynamic general equilibrium, monetary policy

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1 Introduction

Collateralised household debt in EU countries ranges from only 6% of GDP in Greece, up to 65% in the Netherlands and 80% in the United Kingdom, with an average for the EU-15 of 36%. The associated debt contracts are almost always written in nominal terms, have quite significant associated transactions costs and as a consequence are renegotiated relatively infrequently (for example, the UK Council of Mortgage Lenders estimates the average length of a mortgage before refinancing to be 5 years). In many countries a high proportion of the interest payments on this debt are at a fixed rate, with the proportion ranging from 80% in France to 30% in the UK down to zero in Portugal.[1]

Some of these features of the data have been investigated in the literature. A number of papers have examined "financial" models of the business cycle and the role of collateralised debt (for example Bernanke et al., 1999) and some recent papers investigate the role of nominal debt (for example Aoki et al., 2002, Iacoviello, 2005)). However the stickiness of debt contracts, and the observation that fixed rate contracts are common, has still to be addressed. Yet nominal debt stickiness is arguably easier to understand than product price stickiness. A well-known criticism of the standard model of product price stickiness is that the menu costs that ultimately must generate stickiness are unlikely to be large. In the case of debt contracts, in contrast, the costs of adjustment are larger, since typically they will involve re-assessment of collateral or other features of creditworthiness.

In this paper we present a standard new Keynesian dynamic general equilibrium model modified in two key respects to reflect the data features noted above. Firstly, to model sticky debt in a tractable manner, we assume that debt contracts adjust in a process very close to the widely used Calvo (1983) model of product prices. This Calvo adjustment process also allows us to model floating or fixed interest rates on the debt. Secondly these features of debt contracts matter in our model because some consumers face binding credit constraints. To simplify the analysis, we switch off the financial accelerator channel by assuming that the real value of collateral is fixed.

We use this model to examine the impact of these features on the response of an economy to an inflationary shock. Since debt is denominated in nominal terms, an inflationary shock changes its real value. The response of monetary policy to this shock changes the interest payment on the debt. The interac-

[1] Data on stocks of mortgage debt and proportion of fixed rates from Maclennan et al. (1999); data on total UK household debt from Brierley et al. (2002); data on the duration of debt from the Council of Mortage Lenders (2004). Debt renegotiation costs are described in detail for the UK in Miles (2004)

tion of these effects, and the response of financial institutions and consumers, determines the behaviour of the economy following the shock.

Our main results are:

1. Since the direct effect of an inflationary shock on nominal interest rates is strongly contractionary, the response of real interest rates necessary to achieve a given contractionary response is initially small or negative in economies where a significant proportion of debt contracts are at floating rates. Many European countries fall into this category

2. The presence of sluggishly adjusting nominal debt contracts introduces additional, long-lived dynamic responses, as optimising financial institutions bring real debt levels back into line with real collateral, boosting debtors' consumption. As a result real interest rates need ultimately to rise, and stay above steady state for a prolonged period.

3. These new features thus introduce an explanation for a sluggish response of nominal interest rates to inflationary shocks that does not rely on any assumption of interest rate smoothing on the part of the central bank.

4. Fixed rate debt reduces the responsiveness of the economy to monetary policy and shifts the burden of adjustment to a shock onto those with floating rate debt, as well as unconstrained optimising consumers. It therefore requires a more aggressive response of real interest rates to achieve a given contractionary response to inflation.

Both of the novel features of the model are crucial to these results. If the level of indebtedness, or the nature of debt contracts, is to matter at all for the monetary transmission mechanism, some households must face binding credit constraints. Were this not the case, inflationary shocks would simply cause small distributional wealth effects and the only impact of different debt systems would then be at most a second-order one due to the differing risk characteristics of different debt contracts (Campbell and Cocco, 2003). These would completely net out in the perfect risk-sharing framework that underpins the standard representative agent model.

But given our maintained assumption that credit constraints are binding, the nature of debt contracts *does* matter for the transmission mechanism.[2] We assume that debt is collateralised but that there are costs in measuring collateral, so that contracts only adjust infrequently. This affects the response

[2]Since the issue of credit constraints has been widely addressed in the literature (see for example Mankiw, 2000, and Gali et al., 2004) we do not examine it further here.

of the economy in two ways: the response of optimising financial institutions in offering new debt to constrained consumers after an inflationary shock; and the impact of changes in the real value of nominal interest payments on existing debt. The first of these will arise in any system where debt is sticky in nominal terms; the nature of the second effect will depend crucially on whether debt interest payments are made on a floating or fixed rate basis.

Some evidence suggestive of a combined role for nominal debt contracts and credit constraints is the well-known and long-standing empirical correlation between consumption and nominal (as opposed to real) interest rates (e.g., Blinder and Deaton, 1985, Fuhrer and Moore, 1995). Fair (2005) cites this as one of the primary reasons why structural macroeconometric models and unrestricted VAR models (Giordani, 2003) imply that inflationary shocks have effects that are contractionary ex ante, rather than expansionary as implied by standard models. Our model helps to provide a theoretical rationale for these features.

There is much evidence that the monetary transmission mechanism differs between countries (for example Angeloni et al., 2003). Our model implies that institutional features could account for a significant part of these differences. Finding direct empirical support for this is complicated by the constant structural change in European mortgage markets over the past 20 years. However Calza et al. (2006) present some preliminary econometric evidence that supports this hypothesis.

In what follows, section 2 presents the model and describes how it is solved and calibrated. Section 3 describes our results, Section 4 discusses implications for monetary policy, and Section 5 concludes. Appendix A shows the linearised system, and Appendix B contains derivations.

2 The Model

The model is, in most respects, a simple version of the standard dynamic new Keynesian model common in the literature (e.g. Goodfriend and King, 1997). Households consume final goods, supply labour and hold financial assets. Intermediate-goods firms produce differentiated goods which are imperfect substitutes in the production function of final-goods firms. Calvo pricing on the part of intermediate-goods firms gives rise to a new Keynesian Phillips curve. A monetary authority sets the real interest rate as in Clarida et al. (1999).

The non-standard features are the presence of credit-constrained households, and of financial institutions who lend to these households by means of sticky nominal debt contracts.

Note that upper case letters refer to levels, lower case to their log-linearised deviations from non-stochastic steady state values. Symbols without time subscripts refer to steady-state values. Full derivations can be found in Appendix B.

2.1 Households

Households consume, supply labour, lend or borrow and are endowed with a single physical asset whose real value is exogenously given. All households have infinite horizons and rational expectations. Following Iacoviello (2005), we divide households up into two types: type 1 having a higher discount factor than type 2, $\beta_1 > \beta_2$. A household of type j maximises its utility given by:

$$U_t = E_t \sum_{i=0}^{\infty} \beta_j^i u\left(C_{jt+i}, N_{jt+i}\right) \tag{1}$$

where the instantaneous utility function is

$$u\left(C_{jt}, N_{jt}\right) = \frac{C_{jt}^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}} + \upsilon \frac{\left(1-N_{jt}\right)^{1-\frac{1}{\sigma_n}}}{1-\frac{1}{\sigma_n}} \tag{2}$$

where σ_c and σ_n are the elasticities of intertemporal substitution of consumption and of leisure.

We follow the growing convention in the literature of building a monetary model without money (see McCallum, 2001). The maximisation is subject to the real budget constraint[3]

$$A_{jt+1} = \frac{1+R_{jt}}{1+\Pi_{t+1}} \left(A_{jt} + Y_{jt} - C_{jt}\right) + Div_{jt+1} \tag{3}$$

where C_{jt} is consumption, $Y_{jt} = W_{jt}N_{jt} + T_{jt}$ is after-tax labour income (W_{jt} is the real wage, N_{jt} labour supplied and T_{jt} a transfer from government), A_{jt} the household's stock of financial assets at the start of period t. Div_{jt+1} are dividends paid by intermediate firms and financial institutions. $\Pi_{t+1} = P_{t+1}/P_t - 1$ is the rate of inflation between periods t and $t+1$, and R_{jt} the

[3] This can be derived from an asset evolution equation in nominal terms

$$P_{t+1}A_{jt+1} = (1+R_t)P_t\left(Y_{jt} - C_{jt} + A_{jt}\right) + P_{t+1}Div_{jt+1}$$

nominal interest rate, set in period t, and payable in period $t+1$, given by

$$R_{jt} = R_t \text{ if } A_{jt} > 0 \tag{4}$$
$$R_{jt} = R_t^D \text{ if } A_{jt} < 0 \tag{5}$$

where R_t is the short-term interest rate set by the central bank, R_t^D is the rate payable on borrowings which we consider in detail later. While financial assets pay an interest rate that is “safe” in nominal terms since it is set in period t, the real return is uncertain due to inflation.

Households face a further constraint

$$A_{jt} \geq -D_t \tag{6}$$

where D_t is the level of debt which financial institutions are prepared to lend to households (the process determining this level of debt is described in Section 2.2).

Given the difference in subjective discount factors, the existence of a credit constraint in some form is a necessary condition for the existence of a non-stochastic steady state in which type 2s have non-zero consumption. To see this, note that the Euler equation for a household of type j is

$$C_{jt}^{-\frac{1}{\sigma_c}} = \beta_j E_t \left[C_{jt+1}^{-\frac{1}{\sigma_c}} \frac{1+R_{jt}}{1+\Pi_{t+1}} \right] + \chi_{jt} \tag{7}$$

where χ_{jt} is the Lagrange multiplier on the credit constraint. In the steady state we assume below that $R_1 = R_2 = R$ due to competition in credit markets, and if consumption is non-zero for both types of household, (7) becomes

$$1 = \beta_j \left(\frac{1+R}{1+\Pi} \right) + \chi_j \tag{8}$$

which, given $\beta_1 > \beta_2$ cannot hold for both types if credit constraints bind for neither type, since this would imply both χ_1 and χ_2 are zero.[4] However a steady state can exist in which type 1s hold positive assets, so that (6) cannot bind and $\chi_1 = 0$. This implies $R = 1/\beta_1 - 1$ and hence $\chi_2 = 1 - \frac{\beta_2}{\beta_1} > 0$, so credit constraints must bind for type 2s in steady state. We assume that deviations from steady state are sufficiently small that credit constraints bind

[4] Equivalently, any notional steady state in which C_1 was constant and credit constraints did not bind would imply continuously falling C_2 and hence ever-increasing indebtedness which contradicts the assumption of a steady state. Inflation and/or real growth would not affect this result.

for type 2s in all time periods, and never bind for type 1s.

Thus for type 1 (unconstrained) consumers, setting $\chi_{1t} = 0$, equation (7) can be linearised in standard fashion:

$$c_{1t} = E_t c_{1t+1} - \sigma_c \left(r_t - E_t \pi_{t+1} \right) \tag{9}$$

while the binding credit constraint means that type 2 (constrained) households are at a corner solution and are not full optimisers. Their consumption is then given by the budget constraint (3). Linearising (3) gives:

$$c_{2t} = y_{2t} - \delta r_t^D + \delta \left(\Delta d_{t+1} + \pi_{t+1} \right) + \delta \frac{R^D}{1 + R^D} \left(y_{2t} - d_t - r_t^D \right) \tag{10}$$

where $\delta = \frac{D}{C_2}$ is the steady-state debt to consumption ratio of type 2 households. The sum of the first two terms on the right-hand side is linearised disposable income after interest payments. The third term represents the change in nominal debt over the period: if banks offer new debt constrained households accept it to fund consumption. The fourth term is the change in interest payments due to debt deviating from its steady state value.

Although there is some evidence to support it (for example Lawrance, 1991), the assumption of differing discount rates is principally a device to motivate credit constraints. Another way of thinking about credit constraints is within a life cycle model of consumption. For some households the optimal level of consumption, out of current financial wealth and future lifetime earnings, would imply a current level of debt greater than that which a financial institution would be prepared to lend, given that household's collateral. This might apply, for example, for households relatively early in the life cycle, for whom future labour income (that cannot be collateralised) makes up a significant fraction of total wealth.

Each type of household supplies a different type of labour. The linearised first-order conditions for labour supply are standard:

$$n_{jt} = \sigma_n \left(\frac{1 - N_j}{N_j} \right) \left[w_{jt} - \frac{1}{\sigma_c} c_{jt} \right] \tag{11}$$

2.2 Financial institutions

Financial institutions make loans to households. Following Kiyotaki and Moore (1997) we assume that lenders cannot force borrowers to repay their debts unless they are secured. The optimal value of debt is then given by some constant fraction (which we normalise to unity) of households' collateral

K, which we assume constant across time and across households.[5] A financial institution faces costs in deviating from this level. If it lends more than this level, part of the debt is unsecured and there is default risk on this unsecured portion. If it lends less, the cost arises from foregone profit opportunities.

Debt contracts are sticky in nominal terms. To capture this stickiness in a tractable way, we progress by analogy to Calvo's model of the aggregate price level. We assume a constant probability ϕ that any given debt contract will be adjusted in the next period, with complete adjustment towards its optimal value if adjustment does take place.[6] This simple model implicitly assumes that banks face costs, analogous to the "menu costs" discussed in the price adjustment literature, in measuring the creditworthiness of individuals. As a result, households only re-mortgage infrequently.

When deciding the nominal value of a new contract Z_{t+1} at time t the financial institution's problem is

$$\max_{Z_{t+1}} E_t \sum_{i=1}^{\infty} \theta_{t,t+i} (1-\phi)^i \left[\left(R^z_{t+i} - R_{t+i} \right) \frac{Z_{t+1}}{P_{t+i}} - \Omega_{t+i} \right] \tag{12}$$

where $\theta_{t,t+i}$ is the stochastic discount factor of the owners of the financial intermediaries (the unconstrained households):

$$\theta_{t,t+i} = \beta_1^i \left(\frac{C_{1t+i}}{C_{1t}} \right)^{-\frac{1}{\sigma_c}} \tag{13}$$

and financial institutions charge a rate R^z_t on a new contract, and raise floating-rate funds from unconstrained households at the central bank's target rate. Ω_t is a cost, assumed to be quadratic, of deviating from the optimal value of debt

$$\Omega_{t+i} = \varpi \left(\frac{Z_{t+1}/P_{t+i} - K}{K} \right)^2 K \tag{14}$$

where ϖ is a scaling parameter. Since all households have identical collateral this aggregate cost will simply be a scaling of the cost of debt deviating from

[5] This switches off the "financial accelerator". The value of the collateral could be modelled as time-varying by assuming that it provides a constant flow of services and its value is therefore the present value of these services. We investigated this specification and found our key results were unchanged.

[6] As in the original Calvo model, this probability will be treated as exogenous. In reality it will be in part endogenous, but is likely to be constant for any given stable monetary regime. See Graham and Snower (2004) for an example of nominal contracts whose frequency of adjustment varies with the steady state value of inflation.

collateral for any individual household.

Deriving the first-order condition, then linearising gives

$$z_{t+1} - E_t p_{t+1} = E_t \left\{ \frac{[1 - B(1)] F}{B(F)} \pi_{t+1} \right\} \tag{15}$$

where F is the forward shift-operator ($F^i x_t = x_{t+i}$) and $B(F) = 1 - \beta_1 (1 - \phi) F$. This condition gives the expected value of a new real debt contract at time $t+1$ in terms of the expected path of inflation. Note that the scaling parameter in (14), ϖ, has no impact on the dynamics of the system. If ϖ increases, a financial institution's costs and marginal cost increase in the same proportion, but the zero profit condition for financial institutions implies that revenue and hence marginal revenue do too. So ϖ cancels out of the equilibrium condition.

2.2.1 Aggregate debt

At time t a proportion $\phi (1 - \phi)^i$ of financial institutions have reset their contracts i periods earlier and have not had the opportunity to reset them since. So we can sum over all contracts and all financial institutions to obtain the real value of aggregate debt:

$$D_t = \frac{\phi}{P_t} \sum_{i=0}^{\infty} (1 - \phi)^i Z_{t-i} \tag{16}$$

Linearising this gives an expression for the evolution of the nominal value of aggregate debt in terms of the value of individual debt contracts:

$$d_t + p_t = \frac{A(1)}{A(L)} z_t \tag{17}$$

where $A(L) = 1 - (1 - \phi) L$ and L is the lag operator ($L^i x_t = x_{t-i}$).

2.2.2 Floating and fixed rates

The interest rate payable on debt can be either floating or fixed. We do not attempt to model households' decision on whether to hold fixed or floating rate debt, though we will have something to say about the factors influencing the decision.

On floating rate contracts financial institutions charge the nominal interest rate plus a constant spread, which has no impact on the linearised dynamics, so we set it to zero. For fixed rate debt, financial institutions choose the (fairly priced) fixed rate on a particular debt contract, R_t^z, according to a no-arbitrage condition:

$$E_t \sum_{i=1}^{\infty} \theta_{t,t+i} (1-\phi)^i R_t^z Z_t = E_t \sum_{i=1}^{\infty} \theta_{t,t+i} (1-\phi)^i R_{t+i}^D Z_t \tag{18}$$

which when linearised gives[7]

$$r_t^z = E_t \left[\phi r_{t+1} + (1-\phi) r_{t+1}^z \right] \tag{19}$$

The average fixed rate R_t^F payable on fixed rate debt will then be

$$R_t^F = \phi \sum_{i=0}^{\infty} ((1-\phi) L)^i R_t^z \tag{20}$$

Linearising gives

$$r_t^F = \phi r_t^z + (1-\phi) r_{t-1}^F \tag{21}$$

If there is some proportion Ψ of borrowers in fixed schemes the average rate payable on aggregate debt will be

$$r_t^D = \Psi r_t^F + (1-\Psi) r_t \tag{22}$$

where we linearise around a steady state where R is constant so $R^D = R$.

2.2.3 Ownership of financial institutions

Financial intermediaries are owned by unconstrained households. Since these households have access to complete markets any idiosyncratic risk arising from the resetting of the Calvo contracts will be eliminated at the level of the representative household. Competition among financial intermediaries drives expected profits over the life of the contract to zero so, if all debt is floating rate, financial institutions will never earn profits. With some fixed rate debt, profits will be earned on fixed rate contracts after unexpected shocks. This profit is remitted in full to unconstrained households.

[7] Note that in both this expression and (15) the stochastic discount factor cancels out in the linearization, just as in the standard derivation of Calvo price setting.

2.3 Firms

In standard fashion we model two types of firms. Intermediate-goods firms with market power produce Dixit-Stiglitz differentiated goods that are inputs to the production process of final-goods firms who costlessly aggregate them to produce a single consumption good.

Intermediate-goods firms produce differentiated goods by means of a technology in which the labour of the two types of households are imperfect substitutes. The cost minimisation problem is then:

$$\min W_{1t}N_{1t} + W_{2t}N_{2t} \tag{23}$$

subject to the production function

$$Y_t = N_{1t}^{\kappa} N_{2t}^{1-\kappa} \tag{24}$$

giving first-order conditions

$$W_{1t} = MC_t \kappa \frac{Y_t}{N_{1t}} \tag{25}$$

$$W_{2t} = MC_t (1-\kappa) \frac{Y_t}{N_{2t}} \tag{26}$$

where MC_t is real marginal cost.

The output of intermediate-goods firms are imperfect substitutes in the production function of final goods firms which produce a single homogenous consumption good using no other factors of production.

Calvo pricing by intermediate-goods firms allows us to derive a New Keynesian Phillips curve as the solution to an intertemporal profit maximisation problem. But it is well known that the New Keynesian Phillips curve cannot of itself generate observable degrees of inflation persistence, which (acting via the nominal interest rate) plays an important role in our results. To generate inflation persistence we follow Clarida et al. (1999) by adding ad hoc a term in lagged inflation to obtain

$$\pi_t = \eta\pi_{t-1} + \beta_1 (1-\eta) E_t\pi_{t+1} + \gamma mc_t + u_t \tag{27}$$

where the parameter η captures the stickiness of inflation, γ, a function of the underlying parameters, measures the sensitivity of inflation to deviations in real marginal cost mc_t and u_t is a white noise "cost-push" shock.[8] This

[8]Kozicki and Tinsley (2002) generate a New Keynesian Phillips Curve with a backward-

particular form form for the Phillips Curve is not crucial to our results, but our specification has the attractive feature as discussed in Clarida et al. (1999), that in reduced form inflation persistence is endogenous to monetary policy.[9]

Firms, like financial institutions, are owned by unconstrained households. To simplify the model, we assume the existence of a government whose only role is to tax away monopoly profits and remit the proceeds to households lump sum in proportion to their (constant) shares of labour income. Given the Cobb-Douglas technology this has the convenient property that the effective shares of each type in total income are constant and equal to their respective labour shares. We discuss the impact of relaxing this assumption, and allowing profits to flow directly to the firms' owners in Section 4.3.

2.4 Monetary policy

We characterise monetary policy by a simple rule for output

$$y_t = \zeta \pi_t \tag{28}$$

Under standard assumptions, $\zeta < 0$ and the central bank leans against the wind, choosing its policy instrument to contract demand when inflation is above target. The transmission mechanism of the economy then gives a rule for the policy instrument, in this case the real interest rate, of the form:

$$r_t - E_t \pi_{t+1} = \tau (L) \pi_t \tag{29}$$

where $\tau (L)$ is a polynomial in the lag operator.

In the standard model (as in e.g. Clarida et al., 1999) the transmission mechanism is the economy's optimising IS curve and $\tau (L)$ is a constant, so that there is a direct equivalence between a static output rule and a static real interest rate rule. This in turn can be reparameterised as a Taylor Rule for the nominal interest rate under other reasonable assumptions.[10] In our model, the nature of debt contracts determines the transmission mechanism and hence $\tau (L)$ implies additional long-lived dynamics in both real and nominal interest rates. In the short run however, with appropriate calibration of ζ, the resulting

looking component from an optimising framework with higher order lag polynomial adjustment costs of changing prices.

[9] Exogenous inflation persistence could also be introduced straightforwardly with a standard New Keynesian Phillips Curve by setting $\eta = 0$ but allowing u_t to be serially correlated, without significant changes to our results.

[10] Clarida et al. (1999) show that demand shocks and inflation persistence can be incorporated straightforwardly into this framework.

rule for nominal rates very closely resembles the static Taylor Rule.

Our results are not significantly altered if we allow monetary policy to follow a Taylor Rule directly (see section 4.1); but our approach has the advantage of allowing us to focus on distributional consequences of inflationary shocks by making outcomes for output and hence aggregate consumption invariant to the transmission mechanism.

2.5 Identities

Total output is given by

$$Y_t = C_{1t} + C_{2t} \tag{30}$$

and incomes of both types of household exhaust total output so

$$Y_t = Y_{1t} + Y_{2t} \tag{31}$$

While the economy also includes a central bank and firms the model implies that they have zero net financial assets in each period. We can combine (31) with the budget constraints for both types of households (3) and the binding credit constraint (6) to give

$$A_{1t} = D_t \tag{32}$$

The assets of type 1 households will thus equal the liabilities of type 2 households at all times.

2.6 Solution method and calibration

The system of linearised equations describing the economy is shown in Appendix A. We solve this system by the method set out in McCallum (1998).

We calibrate our model on quarterly data with the values shown in table 1 which correspond to those for the UK economy. With the exception of the monetary policy parameter ζ, and Ψ, the proportion of fixed rate contracts, the qualitative nature of our results is insensitive to a wide degree of variation in the assumed parameters.

There is considerable uncertainty as to the quantitative significance of credit constraints: Campbell and Mankiw (1991) find the consumption share of credit constrained consumers to be between 0.2 and 0.65. For simplicity we assume equal labour income shares of one half for both types of consumer. Along with other assumptions on debt and steady state interest rates, this in turn implies that the consumption share of constrained consumers is just under one half.[11] Brierley et al. (2002), using data from the British Household Panel Survey, give debt to annual income ratios ranging from 4 among the lowest income households to 1 among the highest. We take δ, the steady state debt to quarterly consumption ratio of type 2 households, to be 8.[12] We assume that the expected life of debt contracts is given by the average frequency with which individuals re-mortgage. For the debt reset probability, ϕ, we choose 0.05 implying the expected life of a debt contract is $\frac{1}{0.05} = 20$ quarters or 5 years. This is shorter than the notional maturity of most mortgage debt, but reflects the fact that such debt is generally renegotiated on a number of occasions before maturity, most notably on moving house.

We describe monetary policy by an output rule rather than a Taylor rule to make outcomes for output (and aggregate consumption) invariant to the transmission mechanism so allowing us to focus on distributional consequences. To calibrate ζ, the measure of how strongly monetary policy responds to inflation, we first choose the proportion of fixed rate contracts to match that found in the UK ($\Psi = 0.3$). Then, given this transmission mechanism, we choose ζ to give the same impact response of the nominal interest rate as would be implied by a simple Taylor rule of the form $r_t = 1.5\pi_t + .5y_t$ (in section 4.1 we briefly discuss the effect of directly implementing a Taylor Rule). We choose η, the coefficient on lagged inflation in the Phillips curve, to give a realistic degree of inflation persistence given the other parameters.

As to the preferences of the households, the discount rate of unconstrained households β_1 is chosen to give an annual real interest rate of approximately 4%. For β_2 we follow Iacoviello (2005) in choosing a value of 0.95. We set the elasticity of intertemporal substitution of consumption, σ_c to $\frac{1}{2}$ and the elasticity of intertemporal substitution of labour, σ_n to 2. We choose steady state labour supply to match observed hours as in King and Rebelo (1999).

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[11] Given the production function and the impact of the lump-sum tax, the consumption share of *un*constrained consumers is given by $\lambda = \kappa + \frac{R}{1+R}(1-\lambda)\delta$. On our calibration $\lambda = 0.537$

[12] This matches well with UK aggregate data. In 2000 total annual consumption (ABPB) in billions of GBP was 594.8 (blue book table 6.2). Financial Statistics Table 3.1G gives total M4 debt for the household sector (sum of lending on property (AVHG), consumer debt (95.1) and lending to unincorporated businesses (AVHI) as

$$481.8 + 95.1 + 34 = 610.9$$

while total M4 deposits of the personal sector (VSCL) were very similar at 585.6 (ie the household sector is a modest net debtor). Thus annual consumption is very close to total debt, which with a consumption share of constrained consumer of just under one half confirms our choice of $\delta = 8$

Table 1: Calibration		
	Description	**Value**
κ	labour income share of unconstrained households	0.5
δ	steady state debt to consumption ratio of constrained households	8.0
ϕ	debt reset probability	0.05
ζ	coefficient in monetary policy rule	-1.76
η	coefficient on lagged inflation in Phillips curve	0.75
γ	coefficient on marginal costs in Phillips curve (implying goods prices are fixed on average for one year)	0.75
β_1	discount factor of unconstrained households	0.99
β_2	discount factor of constrained households	0.95
σ_c	intertemporal elasticity of substitution for consumption	0.5
σ_n	intertemporal elasticity of substitution for labour	2.0
N	steady state labour supply	0.2

3 Results

In this section we analyze the response to a unit "cost push" shock to the Phillips curve (27) of four model economies: two where all debt is either floating or fixed, and two mixed cases. While we consider the first two cases principally for heuristic purposes, data reported in Maclennan et al. (2000) suggests the purely floating case corresponds closely to Portugal and Finland. In our mixed cases, we consider two economies, one with a relatively low (30%) share of fixed rate debt, which corresponds roughly to the case of the UK, the other with a relatively high (80%) share corresponding roughly to France.

To focus on the distributional consequences of changing the transmission mechanism, we hold all other parameters constant across cases. In particular, the specification of monetary policy is held constant across these cases so that the response of output (and hence, from (30), aggregate consumption) is the same in all cases: a 1.76% reduction on impact (since $\zeta = -1.76$) then a gradually rise back to the steady state as inflation decays.

The dynamics of debt are independent of the proportion of fixed rate debt, and the dynamics of inflation largely so. Before turning to our individual model economies we discuss the processes for debt and inflation that underlie all of them.

3.1 The dynamics of inflation and debt

Inspection of the Phillips curve (27) shows there are potentially two sources for dynamics in inflation: endogenous dynamics given by the parameter η and the dynamics of marginal cost.

To illustrated, consider a simplified version of the model in which the steady state real interest rate is zero. With this assumption, and given our baseline calibration, marginal cost is proportional to output, in deviations $mc_t = \left(\frac{1}{\sigma_c} + \vartheta\right) y_t$ so the Phillips curve can be rewritten as

$$\pi_t = \eta\pi_{t-1} + \beta_1 (1-\eta) E_t\pi_{t+1} + \tilde{\gamma} y_t + u_t \tag{33}$$

where

$$\tilde{\gamma} = \left(\frac{1}{\sigma_c} + \vartheta\right)\gamma \tag{34}$$

Then using the policy rule (28) it is straightforward to show that in this special case inflation follows a first-order autoregressive process.

$$\pi_t = \rho(\eta, \tilde{\gamma}, \zeta)\,\pi_{t-1} + \varepsilon_t \tag{35}$$

where ε_t is a scaling of the "cost-push" shock, u_t, and $\frac{\partial \rho}{\partial \zeta} > 0$: the less monetary policy leans against the wind, the more persistent will be inflation (as in Clarida et al., 1999).

When inflation follows this autoregressive process, the process for the value of a new debt contract (15) can be rearranged to give

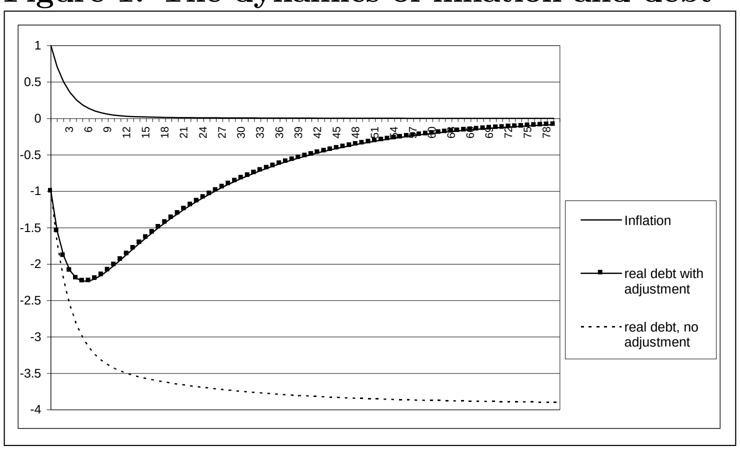
$$z_{t+1} - E_t p_{t+1} = \frac{\beta_1 (1-\phi)\rho^2}{1 - \beta_1 (1-\phi)\rho}\pi_t \tag{36}$$

where the coefficient on inflation is increasing in ρ. If inflation is above its steady state value, the more persistent is inflation, the higher the real value of the contract chosen by financial institutions when they reset the contract's value in nominal terms since the faster it will be eroded.

In the actual model, without the simplifying assumption of a zero steady state real interest rate, extra dynamics are introduced into the system due to the dynamics of consumption affecting those of marginal cost. However these effects are small (since the steady state real interest rate is small) and the process for inflation remains very close to AR(1) in all our calibrations. As a result an (appropriately calibrated) output rule is always close to a static Taylor Rule.

Figure 1 shows the response of real debt to a unit inflationary shock. On impact, real debt falls, continues to fall for 5 quarters, then slowly rises back to its steady state value. This path is the consequence of the interaction of two effects. If the nominal value of debt were constant ($\phi = 0$), the real value of debt would mirror the price level, jumping down by 1% on impact, then falling gradually to a permanently lower long-run value (in the AR(1) case this would be $\frac{1}{1-\rho}$% below its initial value). This is shown by the dotted line in figure 1. But financial institutions reset the nominal value of debt according to the Calvo process described above, which ultimately brings the real value of debt back to its steady state value. As can be seen from figure 1, on impact the 1% inflationary shock causes a 1% fall in the real value of debt since nominal debt is set one period in advance. At first, the inflationary shock's erosion of the real value of debt dominates the debt adjustment process and real debt initially falls. As the inflationary shock decays, the debt adjustment process dominates and the real value of debt gradually returns to its steady state.

Figure 1: The dynamics of inflation and debt[13]



From (32) the assets of unconstrained consumers always equal the debt of constrained consumers. Given that the real value of debt returns to its steady state after a shock (which it must do since real collateral is fixed) the real value of assets must do so too so there are no long run effects.

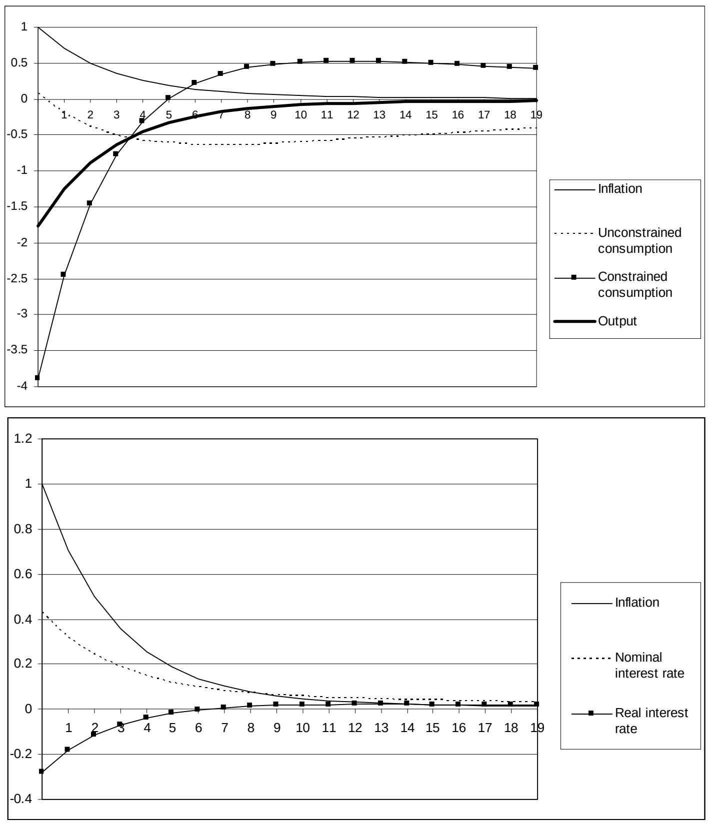
3.2 Floating rate debt

Figure 2 shows the response to an inflationary shock of an economy in which all debt is floating rate ($\Psi = 0$). The burden of adjustment to the inflationary shock falls largely on constrained consumers (indeed on impact almost entirely

[13] Quarters after the shock on the x-axis, deviations from steady state values on the y-axis. Note the x-axis here is longer than in subsequent figures.

so). But the fall in aggregate consumption (in this simple economy identical to the fall in output) is brought about despite an initial fall in real interest rates that is not reversed for nearly two years. To understand the features of the economy that lead to these responses it is useful to start by considering constrained consumption.

Figure 2: All floating rate debt[14]



The consumption of constrained consumers is given by their linearised budget constraint (10) which, since the steady state interest rate is small compared to δ, can be written approximately as:

$$\begin{aligned} c_{2t} &\approx y_{2t} - \delta \left[r_t^D - (\Delta d_{t+1} + \pi_{t+1}) \right] \\ &\approx c_{1t} - \frac{\delta(1-\lambda)}{\lambda} \left[r_t^D - (\Delta d_{t+1} + \pi_{t+1}) \right] \end{aligned} \tag{37}$$

where the second line follows from our calibration and the log linearisation of (30), which implies that $y_{2t} = y_t = \lambda c_{1t} + (1-\lambda) c_{2t}$, where λ is the steady-state

[14] Quarters after the shock on the x-axis, deviations from steady state values on the y-axis.

share of unconstrained consumption.

So there are three effects that determine the path of constrained consumption

1. The output response. Since the central bank sets the real interest rate to achieve a fall in y_t, this reduces constrained consumption in all periods.

2. The response of the debtor rate, r_t^D. With all floating rate debt, the rate paid on nominal debt is (in deviations) equal to the nominal interest rate, which is above its steady state value in all periods after an inflationary shock even when the real rate is below steady state. So this reduces constrained consumption in all periods.

3. The change in nominal debt between periods t and $t+1$ (given by $\Delta d_{t+1} + \pi_{t+1}$). The behaviour of financial institutions means that in the period after the inflationary shock and thereafter, they issue new nominal debt to households to bring the real value of debt back in line with collateral. Since constrained households immediately consume the additional real resources from this new debt, this increases their consumption in all periods.

In the simplest possible case, if constrained consumers had no debt ($\delta = 0$), only the first effect would operate, and their consumption would simply track the output response and hence (from the second line of (37)) that of unconstrained optimising consumers. Since these in turn respond only to real interest rates the economy would in this restricted case have the standard feature that a negative output response to inflation would require a positive response of real interest rates.

With non-zero debt ($\delta > 0$) the response of constrained consumption will differ from that of unconstrained consumption and output, the sign of the difference depending on the relative magnitudes of effect (2) and effect (3), on whether the nominal interest rate rises by more than the rate of increase of nominal debt. While most of the impact of the shock on nominal interest rates decays with inflation the adjustment of real debt is much more prolonged. As a result, as can be seen from the impulse response functions in Figure 2, in the pure floating rate case initially effect (2) dominates; but as the nominal interest rate falls back, effect (3) increasingly dominates, with constrained consumption rising above its steady state after nearly two years before gradually falling back towards it.

This longer-term positive response to an inflationary shock is not a conventional wealth effect due to the fall in the real value of debt shown in Figure 1, since constrained consumers are not intertemporal optimisers. The response is instead driven by the optimising response of financial institutions. The fall in the real value of debt causes them losses in the short term, that they simply pass on to their owners, the unconstrained consumers. Then they optimise by increased lending to bring the real value of debt gradually back in line with collateral. Constrained consumers respond by simply spending these additional funds.

Given the time profile of the response of constrained consumers, the resource constraint implies that unconstrained consumption must respond on impact by less than constrained consumption (indeed it barely falls at all), but, with the real interest rate subsequently rising above steady state, unconstrained consumption falls, and stays below its steady state for long after the impact on inflation itself has disappeared. However, the overall burden of adjustment borne by unconstrained consumers is unambiguously reduced, compared to the cases where there were no credit constraints or constrained consumers had no debt.[15]

The pattern of falling, then rising real interest rates is consistent with the finding (Clarida et al., 1998, among others) that, except in relatively recent years, the historic response of real interest rates to inflation in some countries has been close to zero or even negative. Rather than such behaviour being due either to interest rate smoothing, or breaches of the Taylor Principle, our model suggests that an initial fall in real interest rates may easily arise in economies in which a significant proportion of households are credit constrained and hold floating rate debt.[16] We discuss this issue further below, in Section 4.2.

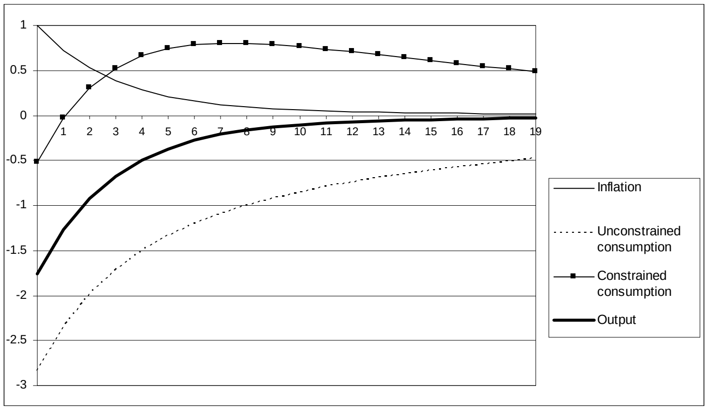
3.3 Fixed rate debt

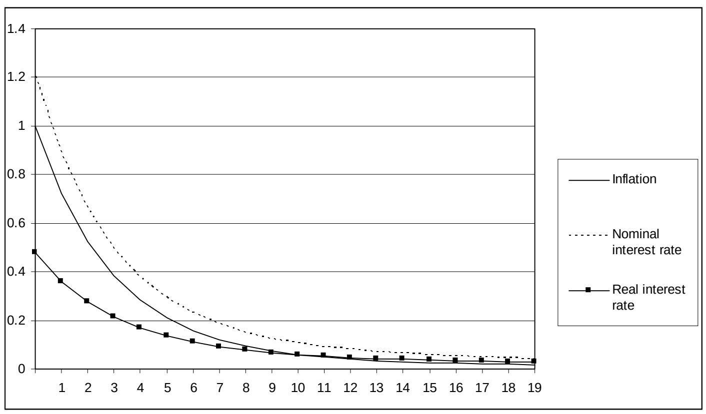
Figure 3 shows the response of the economy to an inflationary shock in an economy with only fixed rate debt ($\Psi = 1$). While the aggregate output response is (by construction) identical to the case with all floating rate debt, the distributional effects on impact are almost precisely reversed. In this case constrained consumption barely falls, so that the impact effect of the fall in output is almost entirely borne by unconstrained consumers.

[15] For a given desired output response to inflation on the part of the central bank, this follows directly from (37) and the identity for output.

[16] It may also be optimal on a utility-based welfare criterion (see Wright, 2004).

Figure 3 : All fixed rate debt[17]





To understand this response, again consider the three determinants of constrained consumption described above. Effects (1) and (3) are close to identical to the floating rate case since the output response and debt responses are determined only by the process for inflation which (for reasons discussed in section 3.1) is barely affected by the transmission mechanism. However effect (2) is very different. Now the rate on debt is equal to the fixed rate r_t^F, given by equation (21) which is very insensitive to changes in nominal rates[18]. This means that effect (2) is dominated by effect (3) in all periods after the shock, so that constrained consumption falls by much less than unconstrained consumption and output on impact, and responds positively after only two quarters. As in the previous case there is a prolonged positive response as financial institutions restore real debt levels in line with collateral.

[17] Quarters after the shock on the x-axis, deviations from steady state values on the y-axis.

[18] It is doubly insensitive because the average fixed rate is a backward moving average of the fixed rate on any given contract, which is itself a forward moving average of short rates.

To achieve a given response of output to inflation, the resource constraint means that unconstrained consumption must follow a steeply rising path, requiring a large impact response of real interest rates.[19]

3.4 Two mixed cases

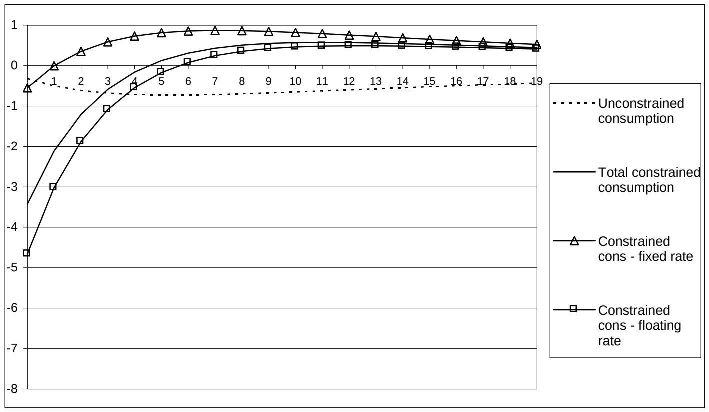
Figures 4 and 5 show the responses of the economy to an inflationary shock in two economies in which there is both floating and fixed rate debt. Figure 4 shows responses with a relatively low proportion of fixed rate debt ($\Psi = 0.3$, roughly corresponding to the UK), Figure 5 when the proportion is relatively high ($\Psi = 0.8$, roughly corresponding to France). Now households can be divided into three types: unconstrained households; constrained households with floating rate debt; and constrained households with fixed rate debt. Since the aggregate consumption response is identical in both cases, and the inflation response almost so these are not shown in Figures 4 and 5.

Since the aggregate output and debt responses are again determined by the path for inflation, and the interest rate on fixed rate contracts is very unresponsive to short-term interest rates, the consumption response of consumers with fixed rate contracts in both mixed cases is virtually identical to the case in which all contracts were assumed to be fixed. The big differences between the two cases therefore arise from the responses of constrained consumers with floating rate contracts, and unconstrained consumers.

With a relatively low share of fixed rate contracts, as in the "UK" case, Figure 4 shows that the overall response of constrained vs. unconstrained consumers is quite similar to that in the pure floating rate case. As a result this case displays the same pattern of an initial fall in the real interest rate that is only reversed after 5 quarters. The only difference is that, with 30% of constrained consumers largely insulated from the impact of the shock, both constrained consumers on floating rate contracts and unconstrained consumers have to bear a larger burden of the aggregate adjustment.

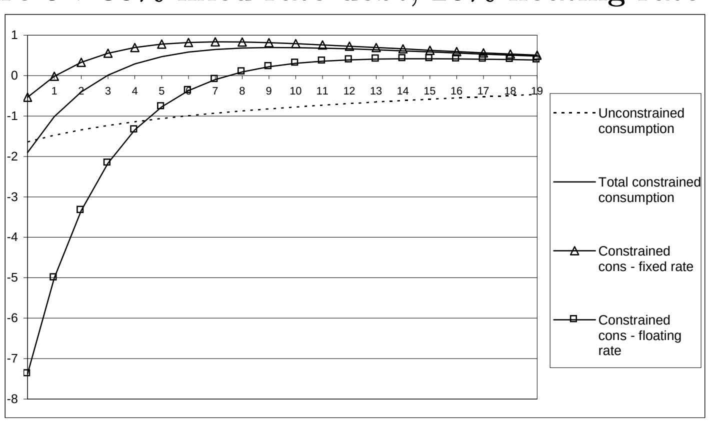
[19] Note that in this case, for a given output response, and hence a given inflation and debt response, the approximation in (37) implies that the relative burden of adjustment on constrained and unconstrained consumers is almost precisely pinned down by δ, the ratio of debt to constrained consumption. An increase in, for example, σ_c, will raise the responsiveness of unconstrained consumers to real interest rates. But this will translate almost wholly into lesser rises in real rates in order to achieve a given output response. It will have no other impact on the economy since constrained consumers are almost entirely insulated from the impact of interest rates given the sluggish response of rates on fixed contracts.

Figure 4 : 30% fixed rate debt, 70% floating rate debt[20]



This latter feature is greatly accentuated in the "French" case shown in Figure 5, in which 80% of constrained consumers are on fixed rate contracts. As a result the burden of adjustment has to fall on the remaining small proportion of constrained consumers on floating rate contracts, and on unconstrained consumers. This in turn requires a distinctly more aggressive response of monetary policy in terms of real interest rates. For floating rate consumers the impact of this response on nominal rates implies over twice as large a fall in their consumption as when all debt contracts are at floating rates.

Figure 5 : 80% fixed rate debt, 20% floating rate debt



This suggests the presence of a network externality effect: as more households join fixed mortgage schemes, so the insurance benefit of a fixed rate scheme increases, in comparison to floating rate schemes. The rise in real interest rates required to satisfy the output rule (28) becomes larger, and as a

[20] Quarters after the shock on the x-axis, deviations from steady state values on the y-axis.

result more of the burden of adjustment is passed on to constrained consumers with floating rate debt. Other things being equal, it might be expected that the existence of such a strong network externality effect would drive floating rate debt out of existence. The fact that we do not observe markets in which fixed rates are completely dominant therefore requires other things to be happening. The most obvious explanation is the existence of a term premium which raises the cost of fixed rate mortgages.

4 Monetary policy

4.1 Taylor Rules vs. Output Rules

We have modelled monetary policy as an output rule so that aggregate effects are held constant as we varied the transmission mechanism in the above sections. How would the results discussed above vary if we modelled monetary policy directly as a Taylor Rule rather than by our assumed output rule? We noted in section 3.1 that the dynamics of inflation and output mean that the output rule we assume is very close to mimicking the impact of a simple static Taylor Rule. But we also showed that as the proportion of fixed rate contracts rose real interest rates needed to respond more aggressively to the inflationary shock in order to achieve a given fall in output. By implication, if instead we held the Taylor Rule coefficients constant the impact on output would be reduced, and hence there would be less of a stabilising impact of monetary policy. The effects are non-trivial: the implied output response on impact in the case of 80% fixed rate debt with fixed Taylor Rule coefficients would be only just over half that in the case of 30% fixed rate debt.[21]

This implies that a monetary authority in a country with a high proportion of floating rate debt (e.g. the UK) should be able to adopt a less aggressive monetary policy in terms of real interest rates than one with a high proportion of fixed rate debt (e.g. the US). However, empirical estimates for Taylor rule coefficients for recent years are roughly the same in these two countries. Our model suggests therefore that the Bank of England is more averse to inflation than the Fed. This inflation aversion has significant distributional consequences, with much of the burden of inflation stabilisation borne by constrained households with floating rate debt.

[21]The entire profile with Taylor Rule coefficients of 1.5 and 0.5 in this case is extremely close to the profile with an output rule as in (28), but with $\zeta = -0.94$, rather than the figure of -1.76 we assume in our baseline calibration.

4.2 Interest rate smoothing

Empirical estimates of Taylor rules, for example Clarida et al. (1998), typically find a large coefficient on the lagged nominal interest rate, with gradual adjustment towards a target real interest rate. A number of explanations of such "interest rate smoothing" can be found in the literature (Woodford, 2003, is one example). With this feature monetary policy may satisfy the Taylor Principle (Woodford, 2001) that real rates should ultimately rise in response to a notional permanent rise in the inflation rate[22] (i.e. in terms of our equation (29) $\tau(1) > 0$), even if real rates fall in the short-term (i.e. $\tau(0) < 0$).

Our results show that this feature can arise without any assumption of interest rate smoothing, solely due to the presence of sticky nominal debt contracts, as long as the share of floating rate contracts is sufficiently high. Notably however the "long-run Taylor Principle" that $\tau(1) > 0$ still applies, albeit with quite long lags.

4.3 Factor shares, stability and uniqueness

Gali et al. (2003) show that the presence of credit constrained consumers (who, unlike in our model, are assumed to have no financial assets or liabilities at all) can quite significantly alter the usual conditions for stability and uniqueness which are otherwise automatically satisfied by the Taylor Rule. This feature arises because in their model, as in ours, the real wage is procyclical due to price stickiness and constrained consumption is driven by the process for labour income. Without debt, and without the assumed role for government in redistributing profits, a very similar feature arises in our model when monetary policy is implemented as a Taylor Rule. However, with the redistribution of profits these effects disappear since shares of both types of consumer in total factor incomes are constant. While the issue of factor income shares is potentially of some importance (and has been largely neglected in recent research since it only arises when there are credit constraints) it is also quite sensitive to other assumptions in the model (for example Fair, 2005, notes that with wage rather than price stickiness real wages are counter-cyclical), so we prefer to base our results on the simpler case of constant factor shares.

An analytical advantage of assuming an output rule rather than Taylor Rule representation of monetary policy is that a sufficient condition for a unique stable solution is that ζ, the parameter in (28) be less than or equal to zero: i.e., that monetary policy "lean against the wind" in terms of output,

[22] It is notional because stabilisation always results in a stationary inflation rate so that permanent shocks to the inflation rate are ruled out.

in response to inflationary shocks. This condition is invariant to the share of credit-constrained consumers as long as we have the redistribution mechanism via taxation such that shares of total factor incomes are constant.

4.4 Policy frontiers

Clarida et al. (1999) note that cost-push shocks lead to a trade-off between inflation and output stabilisation, and that this trade-off is worsened as the persistence of inflation increases. In our model, persistence in inflation arises from the backward looking term in the Phillips curve and the dynamics of marginal cost. We show in Appendix B that marginal cost is given by

$$mc_t = \left(\frac{1}{\sigma_c} + \vartheta\right) y_t + \upsilon \left(c_{2t} - c_{1t}\right) \text{ where } \upsilon = \frac{\lambda - \kappa}{\sigma_c} > 0 \tag{38}$$

As, Ψ, the proportion of fixed rate debt, increases, more of the burden of adjustment falls on unconstrained consumers and less on constrained so $c_{2t} - c_{1t}$ increases in every period. This increases marginal cost in every period and, through the Phillips curve, makes inflation more persistent.

To understand the nature of the trade-off note firstly that for a given level of fixed rate debt, Ψ, if the monetary authority becomes more averse to inflation (ζ becomes more negative) the persistence of inflation is decreased and secondly that the relation between output volatility and inflation volatility can be derived from the output rule (28)

$$\sigma_y^2 = \zeta^2 \sigma_\pi^2 \tag{39}$$

Then consider a particular level of output volatility. As ψ increases this tends to make inflation, and hence output more persistent, so to achieve a given level of output volatility ζ must become less negative. But ζ becoming less negative means from (39) that a higher level of inflation volatility corresponds to each level of output volatility so the trade-off worsens.

Figure 6 : Policy frontiers[23]

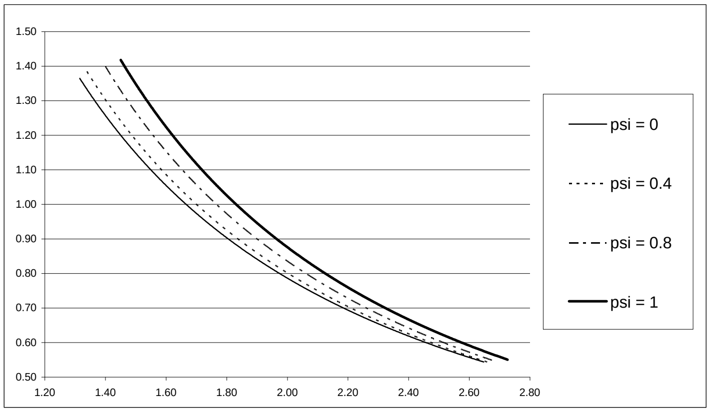


Figure 6 shows how the policy frontiers vary with ψ in the case of an innovation to the Phillips curve with unit standard deviation. As ψ increases the trade-off between inflation volatility and output volatility worsens. For example, in our baseline calibration with all floating rates and $\zeta = -1.76$ the monetary authority achieves output volatility of 1.7 and inflation volatility of 0.97. With all fixed rates, the level of inflation volatility corresponding to this level of output volatility rises to 1.1%, and the policy parameter necessary to achieve it rises to $\zeta = -1.58$. If the central bank is less averse to inflation, inflation is more persistent and the effect of different levels of fixed rate debt becomes bigger. With $\zeta = -1.0$ and all floating rates the central bank achieves output and inflation volatility of 1.31 (the same by (39)). To achieve the same level of output volatility in the all fixed rate cases requires $\zeta = -0.75$ and a volatility of inflation of 1.75.

5 Conclusions

We started this paper with three observations about the data:

(a) Household debt is written in nominal terms

(b) Adjustments to such debt are costly

(c) In many countries a large proportion of interest payments on such debt are at a fixed rate.

In the light of these features of the data we construct a simple dynamic general equilibrium model in which sticky nominal debt contracts play an important role. We use this model to analyze how different debt contracts

[23] Standard deviation of output on the x-axis, of inflation on the y-axis, both as ratios to standard deviation of innovation.

affect the monetary policy transmission mechanism. We show that a simple static Taylor Rule can easily result in a prolonged period in which real interest rates are cut rather than raised in response to an inflationary shock.

Our analysis of a mixed case consisting of households having both fixed and floating rate debt suggests that the cost of adjustment to an inflationary shock is shared very unequally between different types of households, the highest cost being paid by constrained households with floating rate debt. This implies that were a country with predominately floating rate debt (the UK) to join a monetary union consisting of countries with a higher proportion of fixed rate debt (France, Germany, Italy), other things being equal, it would subsequently bear a large proportion of the cost of adjustment to inflationary shocks. It would seem policy is anticipating this potential cost of monetary union, the commissioning of Miles (2004) being one example.

Although we have, following convention, focussed on a positive inflationary shock, the more relevant case in the recent past would arguably be a negative, or disinflationary shock. Our linearised model is of course symmetric, implying that a temporary fall in inflation (or, prospectively, a period of deflation) should boost debtor consumption in a system like the UK where debt is predominantly at floating rates. In the case of deflation this also requires that the value of new debt contracts should fall, if financial institutions are to bring real debt into line with collateral, raising the prospect of some degree of asymmetry if cuts in debt cannot be enforced on recontracting.

The model we have presented is of course highly stylised. To simplify our analysis we chose to switch off the financial accelerator mechanism by assuming collateral is fixed. Endogenising house prices with a model such as that in Aoki et al. (2002) would allow us to relax this assumption. We chose the Calvo style model of debt contracts for its tractability, but, as with the Calvo model of product prices, this is its principal merit. To draw quantitative policy implications from the model it would be necessary to embed it in a more sophisticated framework, with multiple shocks, so that the importance of the role of debt contracts channel could be compared with that of other parts of the transmission mechanism.

A The linearised model

The linearised economy is given by a set of 15 equations in 15 unknowns, y_t, c_{1t}, c_{2t}, π_t, z_t, d_t, r_t, r_t^D, r_t^F, r^z,$n_{1t}, n_{2t}, w_{1t}, w_{2t}, mc_t$

Aggregate demand

The Euler equation for unconstrained consumers:

$$c_{1t} = E_t c_{1t+1} - \frac{1}{\sigma_c}\left(r_t - E_t \pi_{t+1}\right) \tag{40}$$

The debt evolution equation for constrained consumers

$$c_{2t} = y_t - \delta r_t^D + \delta\left(\Delta d_{t+1} + \pi_{t+1}\right) + \delta \frac{R^D}{1+R^D}\left(y_t - d_t - r_t^D\right) \tag{41}$$

where we exploit the fact that the redistributive nature of the tax on profits and lump-sum transfers results in type 2 factor income being in constant proportion to total income, hence $y_{2t} = y_t$.

The resource constraint (30) becomes

$$y_t = \lambda c_{1t} + (1-\lambda)\, c_{2t} \tag{42}$$

Aggregate supply

The Phillips curve with a term in lagged inflation to generate inflation persistence

$$\pi_t = \eta \pi_{t-1} + \beta_1 (1-\eta) E_t \pi_{t+1} + \gamma m c_t + u_t \tag{43}$$

Labour supply curves for type 1 and type 2 households, from (11)

$$\vartheta n_{1t} = w_{1t} - \frac{1}{\sigma_c} c_{1t} \tag{44}$$

$$\vartheta n_{2t} = w_{2t} - \frac{1}{\sigma_c} c_{2t} \tag{45}$$

where $\frac{1}{\vartheta} = \sigma_n \frac{1-N}{N}$

Labour demand for the two types of labour from (25) and (26)

$$w_{1t} = mc_t + y_t - n_{1t} \tag{46}$$

$$w_{2t} = mc_t + y_t - n_{2t} \tag{47}$$

and combining these with the linearised production function gives an expression for the firm's marginal cost

$$mc_t = \kappa w_{1t} + (1 - \kappa) w_{2t} \tag{48}$$

Financial institutions

The optimal nominal value of a new debt contract arising from the optimising behaviour of financial institutions

$$z_{t+1} - E_t p_{t+1} = E_t \left\{ \frac{[1 - B(1)] F}{B(F)} \pi_{t+1} \right\} \tag{49}$$

a process for aggregate debt

$$d_{t+1} + p_{t+1} = \frac{A(1)}{A(L)} z_{t+1} \tag{50}$$

the optimal rate on a new contract

$$r_t^z = E_t \left[\phi r_{t+1} + (1 - \phi) r_{t+1}^z \right] \tag{51}$$

the average rate on a fixed rate contract

$$r_t^F = \phi r_t^z + (1 - \phi) r_{t-1}^F \tag{52}$$

the average rate on all debt

$$r_t^D = \Psi r_t^F + (1 - \Psi) r_t \tag{53}$$

Monetary policy

The monetary policy rule is

$$y_t = \zeta \pi_t \tag{54}$$

A conventional "dynamic IS-LM" model (such as that presented in McCallum, 2001) comprises four equations: a forward-looking IS curve, a Phillips curve, a rule for monetary policy and a resource constraint. Equations (40) to (54) of our model correspond to this, though to enable us to focus on the distributional effects of inflationary shocks we replace McCallum's Taylor rule with an output rule. We add three features to the model. Firstly, the presence of credit constrained consumers whose consumption is given by their budget constraint (44). This heterogeneity in consumption leads to heterogeneity in labour supply so we have to consider the labour market in more detail in equations (44) to (47) Secondly, the presence of a level of a level of debt derived from the optimising behaviour of financial institutions (49) and (50). Thirdly, the behaviour of the interest rate on this debt (51) to (53), again arising from the optimising behaviour of financial institutions.

B Derivations

B.1 Households

B.1.1 Type 1: Credit unconstrained

Households solve the problem

$$\max U = E_t \sum_{i=0}^{\infty} \beta_1^i u\left(C_{1t+i}, N_{1t+i}\right) \tag{55}$$

where

$$U\left(C, N\right) = \frac{C^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}} + \upsilon \frac{\left(1-N\right)^{1-\frac{1}{\sigma_n}}}{1-\frac{1}{\sigma_n}} \tag{56}$$

subject to

$$P_{t+1}A_{t+1} = (1+R_t^D)P_t\left(W_{1t}N_{1t} - C_{1t} + A_t + T_{1t}\right) + P_{t+1}Div_{t+1} \tag{57}$$

The first-order conditions are

$$\begin{aligned}
C_{1t} \quad &: \quad C_{1t}^{-\frac{1}{\sigma_c}} = (1+R_t^D)P_t\Lambda_t && (58)\\
A_{t+1}^N \quad &: \quad \Lambda_t = \beta_1\Lambda_{t+1}R_t^D && (59)\\
N_{1t} \quad &: \quad \upsilon\left(1-N_{1t}\right)^{-\frac{1}{\sigma_n}} = (1+R_t^D)\Lambda_t P_t W_{1t} && (60)\\
A_{t+1} \quad &: \quad C_{1t}^{-\frac{1}{\sigma_c}} = \beta C_{1t+1}^{-\frac{1}{\sigma_c}} R_t \frac{P_{t+1}}{P_t} && (61)
\end{aligned}$$

Linearising the Euler equation gives

$$-\frac{1}{\sigma_c}c_{1t} = -\frac{1}{\sigma_c}c_{1t+1} + r_t - E_t\pi_{t+1} \tag{62}$$

so

$$c_{1t} = E_t c_{1t+1} - \sigma_c\left(r_t - E_t\pi_{t+1}\right) \tag{63}$$

The labour supply curve is

$$\upsilon\left(1-N_{1t}\right)^{-\frac{1}{\sigma_n}} = C_{1t}^{-\frac{1}{\sigma_c}} W_{1t} \tag{64}$$

which when linearised gives

$$\vartheta n_1 = w_{1t} - \frac{1}{\sigma_c} c_{1t} \text{ where } \vartheta_1 = \frac{1}{\sigma_n} \frac{N}{1-N} \tag{65}$$

where w_{1t} is the linearised real wage paid to type 1 households

B.1.2 Type 2: Credit constrained

The budget constraint (57) is

$$P_{t+1} A_{t+1} = \left(1 + R_t^D\right) P_t \left(Y_{2t} - C_{2t} + A_t\right) \tag{66}$$

where R_t^D is the average nominal interest rate payable on nominal debt.

$$D_{t+1} \left(1 + \Pi_{t+1}\right) = \left(1 + R_t\right) \left(C_{2t} - Y_{2t} + D_t\right) \tag{67}$$

Using $\frac{A_t}{P_t} = -D_t$ as the credit constraint binds. In the steady state this gives

$$C_2 = Y_2 - (R - \Pi) D \tag{68}$$

with zero steady state inflation $\Pi = 0$ so

$$C_2 = Y_2 - RD \tag{69}$$

Linearising this

$$(1 + \Pi) \left(d_{t+1} + \pi_{t+1}\right) = C_2 c_{2t} - Y_2 y_{2t} + (1 + R) D \left(r_t + d_t\right) \tag{70}$$

With $\Pi = 0$,divide through by C_2

$$\delta \left(d_{t+1} + \pi_{t+1}\right) = c_{2t} - \frac{Y_2}{C_2} y_{2t} + (1 + R) \delta \left(r_t + d_t\right) \tag{71}$$

From (69)

$$1 + R\delta = \frac{Y_2}{C_2} \tag{72}$$

so

$$\delta \left(d_{t+1} + \pi_{t+1}\right) = c_{2t} - (1 + R\delta) y_{2t} + (1 + R) \delta \left(r_t + d_t\right) \tag{73}$$

$$\begin{aligned} c_{2t} &= y_{2t} - \delta \left(r_t + d_t\right) + \delta \left(d_{t+1} + \pi_{t+1}\right) - R\delta \left(r_t + d_t - y_{2t}\right) \\ &= y_{2t} - \delta r_t + \delta \left(d_{t+1} - d_t + \pi_{t+1}\right) + R\delta \left(y_{2t} - r_t - d_t\right) \end{aligned} \tag{74}$$

Given this level of consumption, constrained households choose their labour supply optimally, and a linearised first-order condition can be derived as for unconstrained households

$$\frac{1}{\sigma_n}\frac{N_2}{1-N_2}n_2 = w_{2t} - \frac{1}{\sigma_c}c_{2t} \tag{75}$$

B.2 Financial institutions

B.2.1 Calvo debt contracts

When deciding the level of a new contract the financial intermediary's problem is

$$\max_{Z_{t+1}} E_t \sum_{i=1}^{\infty} \theta_{t,t+i} (1-\phi)^i \, Prof_{t+i} \tag{76}$$

where $\theta_{t,t+i}$ is the stochastic discount factor of the owners of the financial intermediaries (the unconstrained households)

$$\theta_{t,t+i} = \beta_1^i \left(\frac{C_{1t+i}}{C_{1t}}\right)^{-\sigma_c} \tag{77}$$

Assuming financial intermediaries raise floating-rate funds from unconstrained households at the central bank's target rate profits are given by:

$$Prof_{t+i} = \left(R^z_{t+i} - R_{t+i}\right)\frac{Z_{t+1}}{P_{t+i}} - \Omega_{t+i} \tag{78}$$

and Ω_t is a cost of deviating from optimum value, assumed to be quadratic

$$\Omega_{t+i} = \varpi \left(\frac{Z_{t+1}/P_{t+i} - K}{K}\right)^2 K \tag{79}$$

where ϖ is a constant.

The first order condition is

$$E_t \sum_{1}^{\infty} \theta_{t,t+i} (1-\phi)^i \frac{\left(R^z_{t+i} - R_{t+i}\right)}{P_{t+i}} = \sum_{1}^{\infty} \theta_{t,t+i} (1-\phi)^i \frac{\partial \Omega_{t+i}}{\partial Z_{t+1}} \tag{80}$$

Assuming competition among large numbers of financial intermediaries eliminates expected profits over the life of the contract so

$$E_t \sum_{i=1}^{\infty} \theta_{t,t+i} (1-\phi)^i \, Prof_{t+i} = 0 \tag{81}$$

which, using the definition of profits (79) becomes

$$E_t \sum_{1}^{\infty} \theta_{t,t+i} (1-\phi)^i \, \frac{\left(R^z_{t+i} - R_{t+i}\right)}{P_{t+i}} = \frac{1}{Z_{t+1}} E_t \sum_{1}^{\infty} \theta_{t,t+i} (1-\phi)^i \, \Omega_{t+i} \tag{82}$$

so the first-order condition becomes

$$E_t \sum_{1}^{\infty} \theta_{t,t+i} (1-\phi)^i \left(Z_{t+1} \frac{\partial \Omega_{t+i}}{\partial Z_{t+1}} - \Omega_{t+i} \right) = 0 \tag{83}$$

$$\frac{\partial \Omega_{t+i}}{\partial Z_{t+1}} = \frac{\varpi}{P_{t+i}} \left(\frac{Z_{t+1}}{P_{t+i} K} - 1 \right) \tag{84}$$

so

$$\varpi E_t \sum_{1}^{\infty} \theta_{t,t+i} (1-\phi)^i \begin{pmatrix} Z_{t+1} \frac{1}{P_{t+i}} \left(\frac{Z_{t+1}}{P_{t+i} K} - 1 \right) \\ - \left(\frac{Z_{t+1}/P_{t+i} - K}{K} \right)^2 K \end{pmatrix} = 0 \tag{85}$$

$$\varpi E_t \sum_{1}^{\infty} \theta_{t,t+i} (1-\phi)^i \left(\frac{Z_{t+1}}{P_{t+i}} - K \right) = 0 \tag{86}$$

Noting that in the steady state

$$\theta_{t,t+i} = \beta_1^i \tag{87}$$

and

$$\frac{Z}{P} = K \tag{88}$$

we can linearise this to give

$$\varpi E_t \sum_{1}^{\infty} \beta_1^i (1-\phi)^i \left(z_{t+1} - p_{t+i} \right) = 0 \tag{89}$$

Note that the stochastic discount factor drops out of the expression in exactly the same way it does in the standard derivation of Calvo pricing as long as steady state inflation is assumed to be zero.

This can then be rewritten as

$$\frac{1}{1-\beta_1(1-\phi)} z_{t+1} = E_t\left\{\frac{1}{1-\beta_1(1-\phi)F} p_{t+1}\right\} \tag{90}$$

i.e.

$$z_{t+1} = E_t\left\{\frac{B(1)}{B(F)} p_{t+1}\right\} \tag{91}$$

where $B(F) = 1 - \beta(1-\phi)F$.

To get the expected real value of this contract rearrange (91) to get

$$z_{t+1} = E_t\left\{1 + \frac{B(1) - B(F)}{B(F)} p_{t+1}\right\} \tag{92}$$

so

$$z_{t+1} - E_t p_{t+1} = E_t\left\{\frac{B(1) - B(F)}{B(F)} p_{t+1}\right\} \tag{93}$$

and using the definition $B(F) = 1 - \beta(1-\phi)F$

$$\begin{aligned} B(1) - B(F) &= 1 - \beta(1-\phi) - 1 - \beta(1-\phi)F \\ &= \beta(1-\phi)(F-1) \\ &= (1 - B(1))(F-1) \end{aligned} \tag{94}$$

and

$$(F-1)p_{t+1} = p_{t+2} - p_{t+1} = \pi_{t+2}$$

so

$$\begin{aligned} z_{t+1} - E_t p_{t+1} &= E_t\left\{\frac{1 - B(1)}{B(F)} \pi_{t+2}\right\} \\ &= E_t\left\{\frac{[1 - B(1)]F}{B(F)} \pi_{t+1}\right\} \end{aligned} \tag{95}$$

If inflation follows an autoregressive process

$$\pi_t = \rho\pi_{t-1} + \varepsilon_t \tag{96}$$

then

$$\begin{aligned} z_{t+1} - E_t p_{t+1} &= \left\{ \frac{\beta(1-\phi)}{1-\beta(1-\phi)F} \rho^2 \pi_t \right\} \\ &= \beta(1-\phi)\rho^2 \sum_{i=0}^{\infty} \beta(1-\phi)\pi_{t+i} \\ &= \frac{\beta(1-\phi)\rho^2}{1-\beta(1-\phi)\rho} \pi_t \end{aligned} \tag{97}$$

B.2.2 Aggregate Debt

Level The proportion of banks that still have the contract Z_{t+1-i} set at time $t-i$ will be the proportion that actually reset in that period, ϕ, multiplied by the probability that they haven't reset in the intervening time $\phi(1-\phi)^i$, so summing over all banks the total nominal debt will be:

$$P_{t+1}D_{t+1} = \phi \sum_{0}^{\infty} (1-\phi)^i Z_{t+1-i} \tag{98}$$

Linearising

$$\begin{aligned} d_{t+1} + p_{t+1} &= \frac{\phi}{\phi \sum_0^\infty (1-\phi)^i} \sum_0^\infty (1-\phi)^i z_{t+1-i} \\ &= \frac{1}{\frac{1}{1-(1-\phi)}} \sum_0^\infty (1-\phi)^i z_{t+1-i} \end{aligned} \tag{99}$$

If we define

$$A(L) = 1 - (1-\phi)L \tag{100}$$

(note $A(1) = \phi$) this can be written as

$$d_{t+1} + p_{t+1} = \frac{A(1)}{A(L)} z_{t+1} \tag{101}$$

Rate of change Substituting (95) into (101) gives

$$d_{t+1} + p_{t+1} = \frac{A(1)}{A(L)} \left[E_t \left\{ \frac{[1-B(1)]F}{B(F)} \pi_{t+1} + p_{t+1} \right\} \right] \tag{102}$$

so

$$\begin{aligned}
&A(L)(d_{t+1}+p_{t+1}) \;= \\
&\phi\left[E_t\left\{\frac{[1-B(1)]F}{B(F)}\pi_{t+1}+p_{t+1}\right\}\right] \\
&d_{t+1}+p_{t+1}-(1-\phi)(d_t+p_t) \;= \\
&\phi\left[E_t\left\{\frac{[1-B(1)]F}{B(F)}\pi_{t+1}+p_{t+1}\right\}\right] \qquad (103)
\end{aligned}$$

so

$$\begin{aligned}
A(L)(d_{t+1}+p_{t+1}) &= \phi\left[E_t\left\{\frac{[1-B(1)]F}{B(F)}\pi_{t+1}+p_{t+1}\right\}\right] \\
\Delta d_{t+1}+\pi_{t+1} &= \phi\left[E_t\left\{\frac{[1-B(1)]F}{B(F)}\pi_{t+1}+p_{t+1}\right\}\right]-\phi d_t-\phi p_t \\
&= \phi\left[E_t\left\{\frac{[1-B(1)]F+B(F)}{B(F)}\pi_{t+1}\right\}-d_t\right] \qquad (104)
\end{aligned}$$

Note since $B(F)=1-\beta(1-\phi)F$

$$[1-B(1)]F+B(F)=\beta(1-\phi)F+1-\beta(1-\phi)F=1 \qquad (105)$$

so

$$\Delta d_{t+1}+\pi_{t+1}=\phi E_t\left\{\frac{1}{B(F)}\pi_{t+1}\right\}-\phi d_t \qquad (106)$$

This gives the expected rate of change of nominal debt in terms of inflation and the existing value of debt.

B.2.3 Fixed rate loans

If all debt is floating rate $R^D=R$ and $r_t^D=r_t$.

The fixed rate set at average of expected rates over expected contract duration

$$
\begin{aligned}
r_t^z &= E_t \frac{\sum_{i=1}^{\infty} (1-\phi)^i r_{t+i}}{\sum_{i=1}^{\infty} (1-\phi)^i} \\
&= \frac{\phi}{1-\phi} E_t \sum_{i=1}^{\infty} [(1-\phi) F]^i r_t \\
&= E_t \frac{\phi ((1-\phi) F)}{1-\phi} \sum_{i=0}^{\infty} [(1-\phi) F]^i r_t \\
&= E_t \frac{A(1)}{A(F)} r_{t+1} \qquad (107)
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
[1-(1-\phi) F] r_t^z &= \phi r_{t+1} \\
r_t^z - (1-\phi) r_{t+1}^z &= \phi r_{t+1} \qquad (108)
\end{aligned}
$$

So the average interest rate paid by fixed-rate borrowers is

$$
\begin{aligned}
r_t^F &= \phi r_t^z + \phi (1-\phi) r_{t-1}^z + \phi (1-\phi)^2 r_{t-2}^z + ... \\
&= \phi \sum_{i=0}^{\infty} ((1-\phi) L)^i r_t^z \\
&= \frac{\phi}{1-(1-\phi) L} r_t^z \\
&= \frac{A(1)}{A(L)} r_t^z \qquad (109)
\end{aligned}
$$

Rearranging

$$
r_t^F - (1-\phi) r_{t-1}^F = \phi r_t^z \qquad (110)
$$

So

$$
\begin{aligned}
r_t^F &= \frac{A(1)}{A(L)} E_t \frac{A(1)}{A(F)} r_t \\
&= \frac{\phi^2}{A(L)} E_t \frac{1}{A(F)} r_t \qquad (111)
\end{aligned}
$$

If there is some proportion Ψ of borrowers in fixed schemes then

$$\begin{aligned} r_t^D &= \Psi r_t^F + (1-\Psi)\, r_t \\ &= \left[\Psi \frac{\phi^2}{A(L)} E_t \frac{1}{A(F)} + (1-\Psi) \right] r_t \end{aligned} \tag{112}$$

B.2.4 Profits

Both interest earned and dividends paid by financial institutions are dated $t+1$ in real terms and are realised at the same time as Y_{t+1}, Π_{t+1} etc.

Taking the budget constraint (57) for $j = 2$, setting $R_{2t} = R_t^D$, and, because our assumption of lump-sum taxes mean intermediate firms pay no dividends, and since financial institutions are owned by unconstrained households, $Div_{2t} = 0$

$$(1+\Pi_{t+1})\, A_{2t+1} = \left(1 + R_t^D\right)(A_{2t} + Y_{2t} - C_{2t}) \tag{113}$$

Financial institutions real profits are:

$$Div_{t+1} = \frac{\left(R_t^D - R_t\right)}{1+\Pi_{t+1}} \left[D_t + C_{2t} - Y_{2t}\right] - \Omega_{t+1}^A \tag{114}$$

where Ω_t^A is an aggregate of the cost Ω on all existing contracts. Whether these costs are remitted to type 1 or type 2 households or some combination makes no difference to the following derivation, so arbitrarily assuming they are fully remitted to type 2 households and substituting into asset evolution for $j = 1$ we get

$$\begin{aligned} (1+\Pi_{t+1})\, A_{1t+1} &= (1+R_t)(A_{1t} + Y_{1t} - C_{1t}) + \\ &\quad \left(R_t^D - R_t\right)(D_t + Y_{1t} - C_{1t}) - \Omega_t^A \end{aligned} \tag{115}$$

Adding the process for A_{2t+1}

$$\begin{aligned} (1+\Pi_{t+1})(A_{1t+1} + A_{2t+1}) &= \left(1+R_t^D\right)(A_{1t} + Y_{1t} - C_{1t}) + \\ &\quad \left(R_t^D - R_t\right)(D_t + Y_{1t} - C_{1t}) - \Omega_t^A \\ &\quad + \left(1+R_t^D\right)(A_{2t} + Y_{2t} - C_{2t}) + \Omega_t^A \end{aligned} \tag{116}$$

since $D_t = -A_{2t}$ and $Y_t = Y_{1t} + Y_{2t} = C_{1t} + C_{2t}$

$$(1+\Pi_{t+1})(A_{1t+1} + A_{2t+1}) = \left(1 + R_t^D\right)(A_{1t} + A_{2t}) \tag{117}$$

which gives $A_{1t} = A_{2t}$ for all t if $A = D_0$.

B.3 Firms

Firms' cost minimisation is

$$\min W_{1t}N_{1t} + W_{2t}N_{2t} \tag{118}$$

subject to the production function

$$Y_t = N_{1t}^{\kappa} N_{2t}^{1-\kappa} \tag{119}$$

Write a Lagrangian

$$\mathcal{L} = (W_{1t}N_{1t} + W_{2t}N_{2t}) - MC_t \left(Y_t - N_{1t}^{\kappa} N_{2t}^{1-\kappa}\right) \tag{120}$$

first order conditions

$$W_{1t} = MC_t \kappa \frac{Y_t}{N_{1t}} \tag{121}$$

$$W_{2t} = MC_t \left(1 - \kappa\right) \frac{Y_t}{N_{2t}} \tag{122}$$

So labour incomes are

$$W_{1t}N_{1t} = MC_t \kappa Y_t \tag{123}$$

$$W_{2t}N_{2t} = MC_t \left(1 - \kappa\right) Y_t \tag{124}$$

And profits are:

$$Y_t - W_{1t}N_{1t} - W_{2t}N_{2t} = Y_t \left(1 - MC_t\right) \tag{125}$$

We can write real marginal cost as

$$W_{1t} = MC_t \kappa \left(\frac{N_{2t}}{N_{1t}}\right)^{1-\kappa} \tag{126}$$

$$\frac{N_{1t}}{N_{2t}} = \left(\frac{1}{MC_t \kappa} W_{1t}\right)^{\frac{1}{\kappa - 1}} \tag{127}$$

$$W_{2t} = MC_t \left(1 - \kappa\right) \left(\frac{N_{1t}}{N_{2t}}\right)^{\kappa} \tag{128}$$

$$\begin{aligned} W_{2t} &= MC_t (1-\kappa) \left(\frac{1}{\lambda_t \kappa} W_{1t} \right)^{\frac{\kappa}{\kappa-1}} && (129) \\ MC_t^{\frac{1}{1-\kappa}} &= \left(\frac{1}{\kappa} W_{1t} \right)^{\frac{\kappa}{1-\kappa}} \left(\frac{1}{1-\kappa} W_{2t} \right) && (130) \\ MC_t &= \left(\frac{1}{\kappa} W_{1t} \right)^{\kappa} \left(\frac{1}{1-\kappa} W_{2t} \right)^{1-\kappa} && (131) \end{aligned}$$

Linearising (121), (122) and (131) give respectively

$$w_{1t} = mc_t + y_t - n_{1t} \tag{132}$$

$$w_{2t} = mc_t + y_t - n_{2t} \tag{133}$$

$$mc_t = \kappa w_{1t} + (1-\kappa) w_{2t} \tag{134}$$

B.3.1 Calibrating the supply side

From labour demand (121)

$$W_{1t} N_{1t} = MC_t \kappa Y_t \tag{135}$$

$$W_{2t} N_{2t} = MC_t (1-\kappa) Y_t \tag{136}$$

$$\frac{W_2 N_2}{Y} = (1-\kappa) MC \tag{137}$$

$$\frac{W_1 N_1}{Y} = \kappa MC \tag{138}$$

So, given our calibration, $N_1 = N_2$, and in the steady state

$$MC = \frac{\theta - 1}{\theta} \tag{139}$$

Steady state profits are

$$PROF = (1 - MC) Y \tag{140}$$

$$\begin{aligned} \frac{PROF}{Y} &= 1 - MC \\ &= \frac{1}{\theta} && (141) \end{aligned}$$

By definition

$$\lambda = \frac{C_1}{Y} \tag{142}$$

and if all profits go to unconstrained

$$\xi = \frac{Y_1}{Y} \tag{143}$$

so

$$\frac{Y_1}{C_1} = \frac{\xi}{\lambda} \tag{144}$$

But we also know from the steady state budget constraint

$$\frac{Y_1}{C_1} = 1 - \frac{R}{1+R}\delta^A \tag{145}$$

and

$$\begin{aligned} \delta^A &= \frac{A}{C_1} = \frac{A}{D}\frac{D}{C_2}\frac{C_2}{C_1} && (146) \\ &= \delta\frac{1-\lambda}{\lambda} && (147) \end{aligned}$$

Putting these together

$$1 - \frac{R}{1+R}\frac{1-\lambda}{\lambda}\delta = \frac{\xi}{\lambda} \tag{148}$$

From (135) and (141)

$$\xi = \frac{Y_1}{Y} = \frac{\kappa}{\theta} + \kappa\frac{\theta-1}{\theta}$$

so κ and λ are related by

$$\lambda - \frac{R}{1+R}(1-\lambda)\delta = \xi = \frac{\kappa}{\theta} + \kappa\frac{\theta-1}{\theta} \tag{149}$$

$$\begin{aligned} \kappa &= \frac{\theta}{\theta-1}\left(\lambda - \frac{R}{1+R}(1-\lambda)\delta - \frac{\kappa}{\theta}\right) && (150) \\ &= \lambda - \frac{R}{1+R}(1-\lambda)\delta && (151) \end{aligned}$$

B.3.2 Marginal cost

To solve for the wage of unconstrained households equate labour supply (65) and demand (132)

$$\vartheta\left(mc_t + y_t - w_{1t}\right) = w_{1t} - \frac{1}{\sigma_c}c_{1t} \tag{152}$$

$$w_{1t} = \frac{\vartheta}{\vartheta+1}\left(mc_t + y_t\right) + \frac{1}{\sigma_c\left(\vartheta+1\right)}c_{1t} \tag{153}$$

and similarly

$$w_{2t} = \frac{\vartheta}{\vartheta+1}\left(mc_t + y_t\right) + \frac{1}{\sigma_c\left(\vartheta+1\right)}c_{2t} \tag{154}$$

$$\begin{aligned}
mc_t &= \kappa w_{1t} + (1-\kappa)\, w_{2t} \\
&= \kappa\left(\frac{\vartheta}{\vartheta+1}\left(mc_t + y_t\right) + \frac{1}{\sigma_c\left(\vartheta+1\right)}c_{1t}\right) + \\
&\quad (1-\kappa)\left(\frac{\vartheta}{\vartheta+1}\left(mc_t + y_t\right) + \frac{1}{\sigma_c\left(\vartheta+1\right)}c_{2t}\right) \\
&= \frac{\vartheta}{\vartheta+1}\left(mc_t + y_t\right) + \frac{\kappa}{\sigma_c\left(\vartheta+1\right)}c_{1t} + \frac{(1-\kappa)}{\vartheta+1}c_{2t} \\
\frac{1}{\vartheta+1}mc_t &= \frac{\vartheta}{\vartheta+1}y_t + \frac{\kappa}{\sigma_c\left(\vartheta+1\right)}c_{1t} + \frac{1-\kappa}{\sigma_c\left(\vartheta+1\right)}c_{2t} && (155)
\end{aligned}$$

$$mc_t = \frac{\kappa}{\sigma_c}c_{1t} + \frac{1-\kappa}{\sigma_c}c_{2t} + \vartheta y_t \tag{156}$$

We can write this in terms of consumption as

$$\begin{aligned}
mc_t &= \frac{\kappa}{\sigma_c}c_{1t} + \frac{1-\kappa}{\sigma_c}c_{2t} + \vartheta y_t \\
&= \frac{1}{\sigma_c}\left(\kappa c_{1t} + (1-\kappa)\, c_{2t}\right) + \vartheta y_t \\
&= \frac{1}{\sigma_c}\left(\lambda c_{1t} + (1-\lambda)\, c_{2t} + (\kappa-\lambda)\left(c_{1t} - c_{2t}\right)\right) + \vartheta y_t \\
&= \left(\frac{1}{\sigma_c} + \vartheta\right) y_t + \frac{(\lambda-\kappa)}{\sigma_c}\left(c_{2t} - c_{1t}\right) && (157)
\end{aligned}$$

where, from definition of λ,

$$\lambda - \kappa = \frac{R}{1+R}(1-\lambda)\,\delta > 0$$

In the special case that $R = 0$, which implies $\kappa = \lambda$ then

$$mc_t = \left(\frac{1}{\sigma_c} + \vartheta\right) y_t \tag{158}$$

B.4 The Government

The government fully taxes monopoly profits of intermediate firms and remits them lump sum to households in proportion to their labour income. Profits and labour income exhaust output so

$$Y_t = W_{1t}N_{1t} + W_{2t}N_{2t} + prof_t \tag{159}$$

and from (121), (122) and (125)

$$W_{1t}N_{1t} = MC_t\kappa Y_t \tag{160}$$

$$W_{2t}N_{2t} = MC_t(1-\kappa)\,Y_t \tag{161}$$

$$prof_t = Y_t - W_{1t}N_{1t} - W_{2t}N_{2t} = Y_t(1 - MC_t) \tag{162}$$

Define lump sum taxes

$$T_{1t} = \kappa.prof_t \tag{163}$$

$$T_{2t} = (1-\kappa)\,.prof_t \tag{164}$$

Then after-tax factor incomes are

$$\begin{aligned} Y_{1t} &= W_{1t}N_{1t} + T_{1t} \\ &= \kappa MC_tY_t + \kappa Y_t(1 - MC_t) \\ &= \kappa Y_t \end{aligned} \tag{165}$$

$$\begin{aligned} Y_{2t} &= W_{2t}N_{2t} + T_{2t} \\ &= (1-\kappa)\,MC_tY_t + (1-\kappa)\,Y_t(1 - MC_t) \\ &= (1-\kappa)\,Y_t \end{aligned} \tag{166}$$

so when linearised

$$y_{1t} = y_{2t} = y_t \tag{167}$$

B.5 The process for inflation

If central bank adopts the following rule for output

$$y_t = \zeta \pi_t \tag{168}$$

where, on standard Clarida et al. (1999) assumptions $\zeta < 0$. This implies (via the transmission mechanism) a particular rule for the real interest rate

$$r_t - E_t \pi_{t+1} = \tau_\pi (L) \pi_t \tag{169}$$

The Phillips curve is

$$\pi_t = \beta (1 - \eta) E_t \pi_{t+1} + \eta \pi_{t-1} + \gamma mc_t + u_t \tag{170}$$

In the special case discussed in section (3.1) $R = 0$ and the relation between marginal cost and output is given by (158) we can rewrite the Phillips curve in terms of output:

$$\pi_t = \beta (1 - \eta) E_t \pi_{t+1} + \eta \pi_{t-1} + \tilde{\gamma} y_t + u_t \tag{171}$$

where

$$\tilde{\gamma} = \left(\frac{1}{\sigma_c} + \vartheta \right) \gamma \tag{172}$$

Using the monetary policy rule

$$y_t = \zeta \pi_t \tag{173}$$

to eliminate output gives

$$\pi_t = \frac{\beta (1 - \eta)}{1 - \tilde{\gamma} \zeta} E_t \pi_{t+1} + \frac{\eta}{1 - \tilde{\gamma} \zeta} \pi_{t-1} + \frac{1}{1 - \tilde{\gamma} \zeta} u_t \tag{174}$$

Assume the solution is of the form

$$\pi_t = \rho \pi_{t-1} + \varepsilon_t \tag{175}$$

Substituting this in gives

$$\rho\pi_{t-1} + \varepsilon_t = \frac{\beta(1-\eta)}{1-\tilde{\gamma}\zeta}\rho^2\pi_{t-1} + \frac{\eta}{1-\tilde{\gamma}\zeta}\pi_{t-1} + \frac{1}{1-\tilde{\gamma}\zeta}u_t \tag{176}$$

So ρ is given by

$$\rho = \frac{\beta(1-\eta)}{1-\tilde{\gamma}\zeta}\rho^2 + \frac{\eta}{1-\tilde{\gamma}\zeta} \tag{177}$$

$$\frac{\beta(1-\eta)}{1-\tilde{\gamma}\zeta}\rho^2 - \rho + \frac{\eta}{1-\tilde{\gamma}\zeta}\pi_{t-1} = 0 \tag{178}$$

$$\rho = \frac{1-\tilde{\gamma}\zeta}{2\beta(1-\eta)}\left[1 \pm \sqrt{1 - 4\frac{\beta(1-\eta)\eta}{(1-\tilde{\gamma}\zeta)^2}}\right] \tag{179}$$

Taking the smaller one (the larger is greater than unity)

$$\begin{aligned}\rho &= \frac{1-\tilde{\gamma}\zeta}{2\beta(1-\eta)}\left[1 - \sqrt{1 - 4\frac{\beta(1-\eta)\eta}{(1-\tilde{\gamma}\zeta)^2}}\right] \\ &= \frac{1}{2\beta(1-\eta)}\left[(1-\gamma\zeta) - \sqrt{(1-\tilde{\gamma}\zeta)^2 - 4\beta(1-\eta)\eta}\right]\end{aligned} \tag{180}$$

The error term becomes

$$\varepsilon_t = \frac{1}{1-\tilde{\gamma}\zeta}u_t \tag{181}$$

B.6 Time Line

Start of period t

1. Nominal Interest is paid on asset holdings and savings from the previous period $R_{jt-1}(A_{jt-1} + Y_{jt-1} - C_{jt-1})$

2. Financial institutions pay nominal dividends relating to spread earnings from last period:

$$Div_{jt}.P_t = \left(R^D_{t-1} - R_{t-1}\right) P_{t-1}\left[D_{t-1} + C_{2t-1} - Y_{2t-1}\right]$$

3. This gives nominal assets P_tA_{jt} which are predetermined

4. The shock happens

5. Inflation, output,consumption, nominal debt in $t+1$, the nominal interest rate (dated t, but to be paid in $t+1$), real interest receipts $\frac{1+R_{jt}}{1+\Pi_{t+1}}(A_{jt}+Y_{jt}-C_{jt})$, real dividends Div_{jt}.are simultaneously determined ($Div_{1t} = \left(\frac{R^D_{t-1}-R_{t-1}}{1+\Pi_t}\right)[D_{t-1}+C_{2t-1}-Y_{2t-1}]$). Note real interest receipts and real dividends are *not* predetermined.

6. Time passes

Start of period $t+1$

References

Angeloni, Ignazio, Anil K. Kashyap, Benoit Mojon (eds), (2003), "Monetary Policy Transmission in the Euro Area: A Study by the Eurosystem Monetary Transmission Network", Cambridge University Press

Aoki, Kosuke, James Proudman and Gertjan Vlieghe, (2002), "House prices, consumption, and monetary policy: a financial accelerator approach", Bank of England Working Paper 169.

Bernanke, Ben, Mark Gertler and Simon Gilchrist, (1999), “The financial accelerator in a quantitative business cycle framework”, in Taylor, John and Michael Woodford (eds.), “Handbook of macroeconomics”, Amsterdam. Oxford. Elsevier, 1999.

Blinder, Alan, and Deaton, Angus (1985) “The time series consumption function revisited” Brookings Papers on Economic Activity, pp183-241

Brierley, Peter, Pru Cox and John Whitley, (2002), "Financial pressures in the UK household sector: evidence from the British Household Panel Survey", Bank of England Quarterly Bulletin.

Calvo, Guillermo A, (1983), "Staggered prices in a utility maximising framework", Journal of Monetary Economics 12, pp.383-398.

Calza, Alessandro, Tommaso Monacelli and Livio Stracca, (2006), "Mortgage Markets, Collateral Constraints and Monetary Policy: Do Institutional Factors Matter?", working paper

Campbell, Jeffrey R and Gregory N Mankiw, (1991), "The response of consumption to income: a cross-country investigation", European Economic Review 35 (4), pp.723-756.

Campbell, John Y and Joao F Cocco, (2003), "Household risk management and optimal mortgage choice", Quarterly Journal of Economics 118 (4), pp.1449-1494.

Clarida, Richard, Jordi Gali and Mark Gertler, (1998), "Monetary policy rules in practice, some international evidence", European Economic Review 42, pp.1033-1067.

Clarida, Richard, Jordi Gali and Mark Gertler, (1999), "The science of monetary policy: a new Keynesian perspective", Journal of Economic Literature 37 (4), pp.1661-1707.

Council of Mortgage Lenders (2004), "Market briefing", available at www.cml.org.uk/cml/filegrab/pdf_pub_misc_mbadhoc2004-5.pdf.pdf?ref=3579

Fair, R (2005) "Estimates of the effectiveness of monetary policy" Journal of Money, Credit and Banking, 37(4) pp.645-659

Fuhrer, Jeffrey C and George R Moore, (1995), "Monetary policy trade-offs and the correlation between nominal interest rates and real output", American Economic Review 85 (1), pp.219-239.

Gali, Jordi, Lopez-Salido, J and Valles, J (2004) "Rule of thumb consumers and the design of interest rate rules", forthcoming Journal of Money, Credit and Banking

Giordani, Paolo (2003) "On Modeling the Effects of Inflation Shocks: Comments and Some Further Evidence", Contributions to Macroeconomics: Vol. 3: No. 1, Article 1.

Goodfriend, Marvin and Robert G King, (1997), "The new neo-classical synthesis and the role of monetary policy", NBER Macroeconomics Annual.

Graham, Liam and Dennis J Snower, (2004), "Money growth in dynamic general equilibrium", ECB Working Paper No 412.

Iacoviello, Matteo, (2005), "House prices, borrowing constraints and monetary policy in the business cycle", American Economic Review, 95, 3 (June), pp. 739-764.

King, Robert G and Sergio Rebelo, (1999), "Resuscitating real business cycles", in Taylor, John and Michael Woodford (eds.), "Handbook of macroeconomics", Amsterdam. Oxford. Elsevier, 1999.

Kiyotaki, Nobuhiro and John Moore, (1997), "Credit cycles", Journal of Political Economy 105, pp.211-248.

Kozicki, Sharon and Peter A Tinsley, (2002), "Dynamic specifications in optimizing trend-deviation macro models", Journal of Economic Dynamics and Control 26, pp.1585-1611.

Lawrance, Emily C, (1991), "Poverty and the rate of time preference", Journal of Political Economy 99 (1), pp.54-77.

Maclennan, Duncan, John Muellbauer and Mark Stephens, (1999), "Asymmetries in housing and financial market institutions and EMU", CEPR Discussion Paper 2062.

Mankiw, Gregory N, (2000), "The savers-spenders theory of fiscal policy", American Economic Review 90 (2), pp.120-125.

McCallum, Bennett T, (1998), "Solutions to linear rational expectations models: a compact exposition", Economics Letters 61, pp.143-147.

McCallum, Bennett T, (2001), "Monetary policy analysis in models without money", Federal Reserve Bank of St Louis Quarterly Review 83 (4), pp.145-160.

Miles, David, (2004), "The UK Mortgage Market: Taking a Longer-Term View. Final Report and Recommendations", HMSO Report.

Woodford, Michael, (2001) “The Taylor Rule and optimal monetary policy”, American Economic Reveiw, 91 (2) pp. 232-237

Woodford, Michael, (2003), "Optimal interest-rate smoothing", Review of Economic Studies 70 (4), pp.861-886.

Wright, Stephen, (2004), "Monetary stabilisation with nominal asymmetries", Economic Journal 114 (492), pp.196-222.