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Abstract

The ongoing deregulation of electricity industries worldwide is providing incentives for microgrids, entities that use small-scale distributed generation (DG) and combined heat and power (CHP) applications to meet local energy loads, to evolve independently of the traditional centralised grid in order to provide greater flexibility and energy efficiency to end-use consumers. We examine the impact of start-up costs on the option values and operating schedules of on-site DG installed by a microgrid in the presence of stochastic electricity and fuel prices. We proceed by formulating a stochastic dynamic programme (SDP) for the microgrid that minimises its expected discounted cost over a time horizon and solving it using least-squares Monte Carlo (LSMC) simulation. The expected cost saving that the microgrid realises by having gas-fired DG installed relative to meeting its entire electric load via off-site purchases is the implied option value of DG. Numerical examples indicate that although start-up costs do not significantly lower DG value, they, nevertheless, have a profound impact on the optimal DG operating schedule as the microgrid must incorporate not only current, but also future, expected start-up costs into its current decision-making process as an opportunity cost. As a consequence, the microgrid becomes more hesitant to turn DG units on (off), preferring to wait until the electricity price (natural gas generating cost) exceeds the natural gas generating cost (electricity price) by a significant margin before taking action. We demonstrate that ignoring this tradeoff and proceeding myopically as in the case without start-up costs results in drastically higher expected costs and fewer opportunities to use DG.

Keywords: Distributed generation, stochastic dynamic programming, real options

1 Introduction

A reorganisation of the familiar power system may be taking place. Under the emerging paradigm, a significant fraction of energy conversion from primary fuels to electricity takes place closer to loads, i.e., as distributed generation (DG), than in the current power system, which is characterised by large-scale power generation and electricity delivery at high voltage over long distances (see [12]). Under this new paradigm, the traditional centralised grid still delivers large quantities of energy to end users, but electricity supply is augmented by a new local entity employing DG and enjoying some measure of control independence from the traditional grid. Such an entity will here be referred to as a *microgrid*, while symmetrically, *macrogrid* will be used to describe familiar, traditional electricity supply involving

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large central-station generation, long-distance energy transmission over a network of high-voltage lines, then distribution through medium-voltage radial, or occasionally meshed, networks (see [1] and [4]).

Microgrids promise four major benefits internal to participants in it, namely: 1. the possibility of self-generating electricity at a cost below the delivered cost from the macrogrid;[1] 2. the application of combined heat and power (CHP) technology, which can significantly improve the economics of self generation; 3. the opportunity to tailor the quality of power delivered to suit the requirements of end uses, here called heterogeneous power quality and reliability (PQR); and 4. the more favourable environment potentially created for energy efficiency and small-scale renewable generation investments.

This paper conservatively addresses only the first benefit, namely the possibility of microgrid self-generation at below wholesale macrogrid prices, using a stylised representation of stochastic prices and start-up costs. The additional transmission and distribution costs associated with retail purchases of electricity are ignored, which understates the attraction of DG. And indeed, many other potential societal benefits of DG have been suggested (see [8]). Nonetheless, it is instructive to describe briefly the latter three benefits outlined above, which should motivate the importance of the line of research pursued in this paper towards assessing the potential benefits of microgrids and the incentives necessary to stimulate their development (more detail on these benefit streams can be found in [10]):

1. CHP is likely to occur in microgrids, be it fired by renewable or non-renewable fuel. While the simple cycle efficiency of generation at modern central station power plants will normally exceed any likely competing technology available in small scales, CHP can change the overall efficiency competition considerably, potentially handing microgrids a lower overall carbon footprint (even with similar fuel), as well as lower cost. Since transporting electricity is much more convenient than transporting heat, placing generation where economically attractive heat sinks exist may be a desirable generation configuration, and one that suggests a high degree of generator dispersion.

2. While technical analysis of electricity service PQR can be highly sophisticated, by contrast analysis of the economics of the PQR of end uses is at best rudimentary. If the familiar universal macrogrid PQR is inadequate, backup or power conditioning provision is applied, and often backup is a code requirement, e.g., at hospitals, but otherwise the universal quality is accepted. Matching the PQR delivered to the requirements of the end use can potentially meet our goals at a lower cost than universal PQR. However, delivering heterogeneous PQR clearly poses some potential practical problems as well as benefits.

3. Finally, a philosophical rather than technical point should be made. The decision-maker in a microgrid offers a powerful opportunity to jump some of the hurdles we face in the macrogrid. As the purchaser of fuel inputs, electricity and other, the adopter of generating technologies, and also as possibly the selector of technologies on the demand side, he or she holds a unique vantage point that seems absent in the macrogrid. The alternatives on both demand and supply sides have a chance at being even-handedly considered, and alternatives that have a hard time getting the attention of the macrogrid, such as diffuse renewables, perhaps have a better chance of being chosen; in other words, some of the market failures of the macrogrid might be mitigated.

In this paper, we examine the operation of a hypothetical microgrid that has already installed DG to meet some of its load and purchases electricity from a wholesale spot market as needed. We gain insight into the optimal DG operational policy by taking a real options approach (see [7] and [11]). Specifically, we recognise that owning a flexible DG unit entitles the microgrid with a strip of embedded options to vary its output according to the relative prices of wholesale electricity and natural gas, the fuel on which DG runs. In solving the microgrid's cost-minimisation problem over a fixed time horizon, we obtain not only an optimal operating policy for the DG unit, but also its implied option value, i.e., the maximum amount the microgrid would be willing to pay to rent the unit for the given time horizon. Since we

[1] Note that this implies self generation must beat only the delivered retail electricity price, which is typically two to three times the average wholesale price.

abstract from transmission and distribution costs, this latter estimate is a lower bound for the DG unit's value. We also do not account for other DG benefits, such as CHP, or allow for optimisation over its investment decision (see [14], [15], and [16] for a more comprehensive analysis in a deterministic setting). Rather, our focus here is on determining the impact of stochastic electricity and fuel prices as well as DG start-up costs on the value of the DG unit and its operational policy.

The structure of this paper is as follows:

- Section 2 formulates the microgrid's problem and outlines the simulation algorithm to solve it
- Section 3 explores how the option values and operating schedules of DG units are affected by start-up costs
- Section 4 summarises the findings of this paper and offers directions for future research

2 Problem Formulation

2.1 Microgrid's Decision-Making Problem

We consider the decision-making problem over a time horizon, T (in years), of a microgrid that has installed two on-site gas-fired DG units of capacity G_L and G_S (both in MW) to meet a constant load, D (in MW), where $G_L > G_S$ and $D > G_L + G_S$, and also retains the option to purchase electricity from the spot market. The natural logarithm of the electricity spot price during hour t is X_t (in \$/MWh), while that of the natural gas price is Y_t (in \$/MMBTU). These two follow (positively correlated) stochastic processes as some central-grid electricity is generated using natural gas as an input fuel. Furthermore, the heat rates of the two DG units are H_L and H_S (both in MMBTU/MWh), where $H_L < H_S$, i.e., the larger DG unit is more efficient than the smaller one. Finally, the start-up costs of the two DG units are U_L and U_S (in \$/h per start), which incorporate the variable operating and maintenance (O&M) costs of the DG units, thereby taking into account the additional wear associated with frequent changes in operating status.

Since both electricity and natural gas prices are stochastic, the microgrid's decision each period is to determine whether or not to use DG. Specifically, during each period t, the microgrid could be in one of four possible operating states, $m_t \in M = \{E, (S, E), (L, E), (L, S, E)\}$ defined as follows:

- $m_t = E$: both DG units are turned off, and the entire load is met via off-site purchases of electricity
- $m_t = (S, E)$: the large DG unit is turned off, the small DG unit is operating at its rated capacity, and the residual load is met via off-site purchases of electricity
- $m_t = (L, E)$: the small DG unit is turned off, the large DG unit is operating at its rated capacity, and the residual load is met via off-site purchases of electricity
- $m_t = (L, S, E)$: both DG units are operating at their rated capacities, and the residual load is met via off-site purchases of electricity

Correspondingly, the microgrid's operating decision at time t is $z_t \in M$.

Based on this structure, the microgrid's minimum expected discounted cost to go at time t given state m_t and price vector (X_t, Y_t) is $V_t(m_t; X_t, Y_t)$. If $C_t(z_t, m_t; X_t, Y_t)$ is the cost in period t associated with making decision z_t while in state m_t and facing price vector (X_t, Y_t), then for $t = 1, 2, \ldots, K-1$:

$$V_t(m_t; X_t, Y_t) = \min_{z_t \in M} \{C_t(z_t, m_t; X_t, Y_t) + \beta E_t[V_{t+1}(m_{t+1} = z_t; \tilde{X}_{t+1}, \tilde{Y}_{t+1})]\} \tag{1}$$

Here, $\beta = e^{-r\Delta t}$ is the one-period discount factor, where r is the annual risk-adjusted interest rate and $\Delta t = T/K$ is the length of the time period in years (the total time horizon divided by the number of decision-making periods). The cost function, $C_t(z_t, m_t; X_t, Y_t)$, is specified as follows for $t = 1, 2, \ldots, K$:

$$C_t(z_t, m_t = E; X_t, Y_t) = \begin{cases} z_t = E: & e^{X_t} D\Delta t\nu \\ z_t = (S,E): & H_S e^{Y_t} G_S \Delta t\nu + U_S \Delta t\nu + e^{X_t}(D - G_S)\Delta t\nu \\ z_t = (L,E): & H_L e^{Y_t} G_L \Delta t\nu + U_L \Delta t\nu + e^{X_t}(D - G_L)\Delta t\nu \\ z_t = (L,S,E): & H_L e^{Y_t} G_L \Delta t\nu + U_L \Delta t\nu + H_S e^{Y_t} G_S \Delta t\nu + U_S \Delta t\nu \\ & +e^{X_t}(D - G_L - G_S)\Delta t\nu \end{cases} \tag{2}$$

$$C_t(z_t, m_t = (S,E); X_t, Y_t) = \begin{cases} z_t = E: & e^{X_t} D\Delta t\nu \\ z_t = (S,E): & H_S e^{Y_t} G_S \Delta t\nu + e^{X_t}(D - G_S)\Delta t\nu \\ z_t = (L,E): & H_L e^{Y_t} G_L \Delta t\nu + U_L \Delta t\nu + e^{X_t}(D - G_L)\Delta t\nu \\ z_t = (L,S,E): & H_L e^{Y_t} G_L \Delta t\nu + U_L \Delta t\nu + H_S e^{Y_t} G_S \Delta t\nu \\ & +e^{X_t}(D - G_L - G_S)\Delta t\nu \end{cases} \tag{3}$$

$$C_t(z_t, m_t = (L,E); X_t, Y_t) = \begin{cases} z_t = E: & e^{X_t} D\Delta t\nu \\ z_t = (S,E): & H_S e^{Y_t} G_S \Delta t\nu + U_S \Delta t\nu + e^{X_t}(D - G_S)\Delta t\nu \\ z_t = (L,E): & H_L e^{Y_t} G_L \Delta t\nu + e^{X_t}(D - G_L)\Delta t\nu \\ z_t = (L,S,E): & H_L e^{Y_t} G_L \Delta t\nu + H_S e^{Y_t} G_S \Delta t\nu + U_S \Delta t\nu \\ & +e^{X_t}(D - G_L - G_S)\Delta t\nu \end{cases} \tag{4}$$

$$C_t(z_t, m_t = (L,S,E); X_t, Y_t) = \begin{cases} z_t = E: & e^{X_t} D\Delta t\nu \\ z_t = (S,E): & H_S e^{Y_t} G_S \Delta t\nu + e^{X_t}(D - G_S)\Delta t\nu \\ z_t = (L,E): & H_L e^{Y_t} G_L \Delta t\nu + e^{X_t}(D - G_L)\Delta t\nu \\ z_t = (L,S,E): & H_L e^{Y_t} G_L \Delta t\nu + H_S e^{Y_t} G_S \Delta t\nu \\ & +e^{X_t}(D - G_L - G_S)\Delta t\nu \end{cases} \tag{5}$$

Equations 2 through 5 describe the cost function in each of the four states given the decision taken in that state, where ν is a factor that converts the length of the time period Δt into hours from years. Note that since turning off either DG unit is costless, the first element of each equation is identical. The only differences are in the second, third, and fourth elements, where start-up costs, U_L and U_S, are added as needed depending on the operating state. Using equations 1 through 5, the microgrid's stochastic dynamic programme (SDP) may be written and solved subject to the following terminal condition:

$$V_K(m_K; X_K, Y_K) = \min_{z_K} \{C_K(z_K, m_K; X_K, Y_K)\} \tag{6}$$

The minimised expected discounted cost of meeting the microgrid's energy needs is $\beta V_1(m_1 = E; X_1, Y_1)$.

In the absence of start-up costs, the microgrid's optimal decision-making rule is a myopic one: simply use the cheaper source of energy without taking into account future states. In this case, owning DG is similar to holding a strip of cross-commodity European options[2] for each hour of the year. Thus, the option to use DG is exercised as long as the per MWh cost of DG generation is less than the price of electricity in the spot market. Consequently, the implied option value of DG may be determined by summing (or by integrating in a continuous-time case) the hourly option values over the entire year (see [6]). In case of path-dependent features, closed-form expressions for option values may not available, thereby necessitating the use of Monte Carlo simulation (see [2]).

While simulation is an efficient procedure for pricing those European options without closed-form solutions, its drawback is that since it is a forward-induction procedure, simulation is not always applicable to options for which optimal exercise policies are needed in advance, such as early-exercise or compound

[2]European options may be exercised only at the date of expiration. By contrast, American options may be exercised at any time up to and including the expiration date.

options. [3] Indeed, for many real options, which have such features, the current state of the system does not include information on the expectation of future events. For example, standard simulation would not be applicable to the microgrid's problem if its DG has start-up costs or operating constraints because then the current state of the DG units alters the payoff structure of any remaining options to generate. It is, therefore, necessary to estimate the impact of current decisions on future cash flows given the current state. Typically, a backward-induction procedure, based on a lattice that discretises the underlying securities' stochastic processes, is used to price such real options (see [3], [5], [7], and [13]). However, if either the number of underlying securities is large or the stochastic processes are too complex to be discretised adequately, then such lattice-based methods are computationally intractable.

In order to resolve this dilemma, a recently developed procedure, known as least-squares Monte Carlo (LSMC) simulation, prices early-exercise options by first generating a large number of sample paths for the underlying securities' prices and then estimating a conditional expectation payoff function via least-squares cross-sectional regressions (see [9]). Specifically, at each time step, the cash flows from continuing to hold on to the option are regressed on a function of current security prices to yield estimated response parameters. These may then be used to estimate a payoff continuation function conditional on current security prices. For a large number of states or underlying stochastic prices, LSMC simulation may be generalised to yield, for example, an estimated conditional continuation function for each state by regressing the cash flows from continuation in each state on a function of all the prices (see [17] and [18] for applications of LSMC simulation to real options). We proceed in section 2.2 to outline the LSMC simulation algorithm needed to solve the microgrid's SDP.

2.2 LSMC Simulation Algorithm

The first step in solving the microgrid's SDP (equations 1 through 6) using LSMC simulation is to generate N sample paths for the electricity and natural gas prices, with $(s_t^{(n)}, w_t^{(n)})$ representing the period t prices for sample path n. In this context, the decision variable is $z_t^{(n)}$, where $\mathbf{z}$ is a $N \times K$ matrix and $\underline{z}_t$ is a $N \times 1$ vector. The value and continuation functions are defined and discussed as follows:

- $V_t(m_t; s_t^{(n)}, w_t^{(n)})$: minimum expected discounted cost to go in period t given state m_t along sample path n (also known as the *value function*)
- $\Phi_t(s_t^{(n)}, w_t^{(n)}; m_t, z_t^{(n)}) \equiv E_t[V_{t+1}(z_t^{(n)}; s_{t+1}^{(n)}, w_{t+1}^{(n)})]$: expected continuation value in period t for sample path n given state m_t and operational decision $z_t^{(n)}$
- $\hat{\Phi}_t(s_t^{(n)}, w_t^{(n)}; m_t, z_t^{(n)}) = \underline{f}(s_t^{(n)}, w_t^{(n)})\underline{\hat{b}}_t(m_{t+1} = z_t^{(n)})$: estimated continuation value in period t for sample path n given state m_t and the decision taken is $z_t^{(n)}$, where the estimated response parameter vector at period t given state m_{t+1} in period $t+1$ from a cross-sectional, least-squares regression of period $t+1$ value functions in state m_t on a function of period t prices is $\underline{\hat{b}}_t(m_{t+1}) = [(\mathbf{f}(\underline{s}_t, \underline{w}_t))^T(\mathbf{f}(\underline{s}_t, \underline{w}_t))]^{-1}(\mathbf{f}(\underline{s}_t, \underline{w}_t))^T\underline{V}_{t+1}(m_{t+1}; \underline{s}_{t+1}, \underline{w}_{t+1})$

Here, since $\mathbf{V}$ is a $N \times K \times |M|$ tensor, $\underline{V}_t(m_t; \underline{s}_t, \underline{w}_t)$ is a $N \times 1$ vector, corresponding to the value vector during period t in state m_t given the $N \times 2$ price vector $(\underline{s}_t, \underline{w}_t)$. Similarly, $\hat{\mathbf{\Phi}}$ is a $N \times K \times |M| \times |M|$ tensor, with the last two dimensions having $|M|$ possible values each since both the number of possible states and number of available decisions are equal to the cardinality of the state space. Therefore, the expected discounted cost to go of the microgrid during period t given state m_t is:

$$V_t(m_t; s_t^{(n)}, w_t^{(n)}) = \min_{z_t^{(n)}}\{C_t(z_t^{(n)}, m_t; s_t^{(n)}, w_t^{(n)}) + \beta E_t[V_{t+1}(z_t^{(n)}; s_{t+1}^{(n)}, w_{t+1}^{(n)})]\} \tag{7}$$

[3] Compound options entitle the holder to gain access to embedded flexibility within the project, e.g., if the microgrid installs a DG unit and then has the additional option to upgrade to incorporate CHP applications, then the initial DG investment is a compound real option. For this reason, compound options are often referred to as being "options on options."

where $C_t(z_t^{(n)}, m_t; s_t^{(n)}, w_t^{(n)})$ is the cost associated with taking decision $z_t^{(n)}$. The dimensions of $\underline{f}(s_t^{(n)}, w_t^{(n)})$ and $\hat{\underline{b}}_t(m_t)$ depend on the number of elements in $\underline{f}$. Following [9], we let $\underline{f}(s_t^{(n)}, w_t^{(n)})$ be a 1×6 vector setting $\underline{f}(s_t^{(n)}, w_t^{(n)}) = [\ 1 \quad s_t^{(n)} \quad w_t^{(n)} \quad {s_t^{(n)}}^2 \quad {w_t^{(n)}}^2 \quad s_t^{(n)} w_t^{(n)}\ ]$, which implies that $\hat{\underline{b}}_t(m_t)$ is a 6×1 vector. Consequently, $\mathbf{f}(\underline{s}_t, \underline{w}_t)$ is a $N \times 6$ matrix.

As indicated in Figure 1, the LSMC simulation approach proceeds by first generating N sample paths for the stochastic electricity and natural gas prices. Next, in line 2, a function of the prices at each time period t and sample path n is constructed as described in Section 3. In line 3, the terminal condition for the value function is set: the decision is simply to minimise the immediate cost. Then, after the estimated continuation function is initialised to zero in line 4, the main recursion begins proceeds backwards in time as follows:

- Line 7: the response parameter vector for period t and future state m_{t+1} is estimated by least-squares regression of period $t+1$ value functions in state m_{t+1} on the function of the period t prices
- Line 9: the estimated continuation function at period t for state m_t given the decision taken is $z_t^{(n)}$ is obtained by multiplying the function of the period t prices by the response parameter vector for time period t and future state $m_{t+1} = z_t^{(n)}$
- Line 10: the optimal operational decision at time t is that which minimises the expected discounted cost to go using the *estimated* continuation function
- Line 11: the value function is updated by using the cost associated with optimal decision $z_t^{(n)*}$ and the *actual* future value
- Line 15: the minimum expected cost is simply the average of the N costs at time 1 in initial state E

In particular, line 10 uses estimated continuation functions to make DG operating decisions, where the immediate cash outflow from the decision is simply the immediate cost of meeting the load. Therefore, the optimal decision is to minimise the cost from making a decision plus the estimated continuation value of proceeding optimally thereafter from the future state in period $t+1$. Note that while estimated continuation functions are used to decide DG operation, the value functions are recursively updated by employing *actual* continuation functions. Hence, by working recursively backwards through all of the time periods, the average minimised value of the microgrid's operating cost may be determined as the average over all N sample paths of costs in period 1 when starting from a position in which both DG units are off, i.e., starting with the terminal condition of line 2 in Figure 1, the LSMC simulation procedure works backwards, updating $V_t(m_t; s_t^{(n)}, w_t^{(n)})$ using the recursion in equation 7 until the answer is obtained as in line 15 of Figure 1.

3 Numerical Examples

In this section, we examine the properties of the microgrid's DG system via hourly analysis over one test year. We assume that $r = 0.045$ per annum, $T = 1$ year, $\Delta t\nu = 1$ hour, $D = 1$ MW, $G_L = 0.50$ MW, and $G_S = 0.20$ MW. Using the data from [16] on 500 kW and 200 kW reciprocating engines, we use $H_L = 10.3$ MMBTU/MWh and $H_S = 11.1$ MMBTU/MWh. There are other costs associated with DG, such as turnkey and variable O&M costs. We do not consider the former explicitly since the microgrid is assumed to have installed DG already. Nevertheless, the implied option value of the DG units from our analysis could provide a lower bound on the annuity the microgrid should pay to install DG. As for the variable O&M costs, we incorporate them into the start-up costs as more frequent operational changes increase the wear on the DG equipment. From the data in [16], the variable O&M costs are \$0.012/kWh

`Generate` $\mathbf{s}, \mathbf{w}$

$\underline{f}(s_t^{(n)}, w_t^{(n)}) = [\ 1 \quad s_t^{(n)} \quad w_t^{(n)} \quad {s_t^{(n)}}^2 \quad {w_t^{(n)}}^2 \quad s_t^{(n)} w_t^{(n)}\ ]$, $n = 1, 2, \ldots, N$, $t = 1, 2, \ldots, K$

$V_K(m_K; s_K^{(n)}, w_K^{(n)}) = \min_{z_K^{(n)}} \{C_K(z_K^{(n)}, m_K; s_K^{(n)}, w_K^{(n)})\}$, $m_K, z_K^{(n)} \in M$, $n = 1, 2, \ldots, N$

$\hat{\Phi}_t(s_t^{(n)}, w_t^{(n)}; m_t, z_t^{(n)}) = 0$, $t = 1, 2, \ldots, K-1$, $m_t, z_t^{(n)} \in M$, $n = 1, 2, \ldots, N$

`For` $t = K-1, \ldots, 1$

  `For` $m_t \in M$

    $\underline{\hat{b}}_t(m_{t+1}) = [(\mathbf{f}(\underline{s}_t, \underline{w}_t))^T (\mathbf{f}(\underline{s}_t, \underline{w}_t))]^{-1} (\mathbf{f}(\underline{s}_t, \underline{w}_t))^T \underline{V}_{t+1}(m_{t+1}; \underline{s}_{t+1}, \underline{w}_{t+1})$

    `For` $n = 1, 2, \ldots, N$

      $\hat{\Phi}_t(s_t^{(n)}, w_t^{(n)}; m_t, z_t^{(n)}) = \underline{f}(s_t^{(n)}, w_t^{(n)}) \underline{\hat{b}}_t(m_{t+1} = z_t^{(n)})$;

      $z_t^{(n)*} = \arg\min_{z_t^{(n)} \in M} \{C_t(z_t^{(n)}, m_t; s_t^{(n)}, w_t^{(n)}) + \beta \hat{\Phi}_t(s_t^{(n)}, w_t^{(n)}; m_t, z_t^{(n)})\}$;

      $V_t(m_t; s_t^{(n)}, w_t^{(n)}) = C_t(z_t^{(n)*}, m_t; s_t^{(n)}, w_t^{(n)}) + \beta V_{t+1}(z_t^{(n)*}; s_{t+1}^{(n)}, w_{t+1}^{(n)})$;

    `End`

  `End`

`End`

`Min Cost` $= \frac{\sum_{n=1}^{N} V_1(E; s_1^{(n)}, w_1^{(n)})}{N}$;

Figure 1: Solution Procedure for LSMC Simulation

and \$0.015/kWh for the large and small DG units, respectively. Since each DG unit may be operated either at full capacity or not at all, the hourly variable O&M costs are obtained by multiplying the per kWh costs by the respective capacities. This yields $U_L = \$6$/h and $U_S = \$3$/h per start, which may be thought of as the *additional* O&M expense that must be borne due to starting up a DG unit.

Finally, we assume that the microgrid functions in an idealised deregulated market, where all electricity and natural gas must be purchased at spot prices.[4] Following [7], we assume that short-term evolution of the natural logarithms of both electricity and natural gas prices can best be described by correlated mean-reverting Ornstein-Uhlenbeck (OU) processes. Specifically:

$$dX_t = \kappa_X(\theta_X - X_t)dt + \sigma_X dS_t \tag{8}$$

$$dY_t = \kappa_Y(\theta_Y - Y_t)dt + \rho\sigma_Y dS_t + \sqrt{1-\rho^2}\sigma_Y dW_t \tag{9}$$

Here, for process i, θ_i is the long-term mean, κ_i is the rate of mean reversion, σ_i is the annualised volatility for process i, and ρ is the instantaneous correlation coefficient between $\{X_t, t \geq 0\}$ and $\{Y_t, t \geq 0\}$. Furthermore, $\{S_t, t \geq 0\}$ and $\{W_t, t \geq 0\}$ are independent standard Brownian motion processes. The OU processes in equations 8 and 9 may be simulated as follows using two independent standard normal random variables ϵ_X and ϵ_Y:

$$X_{t+\Delta t\nu} = X_t + \kappa_X(\theta_X - X_t)\Delta t + \sigma_X \epsilon_X \sqrt{\Delta t} \tag{10}$$

$$Y_{t+\Delta t\nu} = Y_t + \kappa_Y(\theta_Y - Y_t)\Delta t + \sigma_Y \rho \epsilon_X \sqrt{\Delta t} + \sqrt{1-\rho^2}\sigma_Y \epsilon_Y \sqrt{\Delta t} \tag{11}$$

[4]In reality, most microgrids would face stable and relatively high electricity tariff rates from incumbent utilities. For example, in the service territory of Pacific Gas and Electric (PG&E) in California, the energy charge in year 2004 was around \$0.10/kWh, along with a demand charge of \$14.35/kW during peak hours and a monthly fixed fee of \$175 (see [16]). Consideration of these additional costs would probably make the DG units more attractive than they are in the present analysis.

Using the data from [7] (reproduced in Table 1) and initial prices of \$21.7/MWh and \$3.16/MMBTU for electricity and natural gas, respectively, we generate $N = 1000$ sample paths. The effective cost of generation from the DG units is obtained by multiplying the natural gas price by the appropriate heat rate. An example of the simulated paths is shown in Figure 2.

θ_X	θ_Y	σ_X	σ_Y	κ_X	κ_Y	ρ
3.2553	0.87	0.79	0.60	3	2.25	0.30

Table 1: Parameter Data for Correlated OU Price Processes

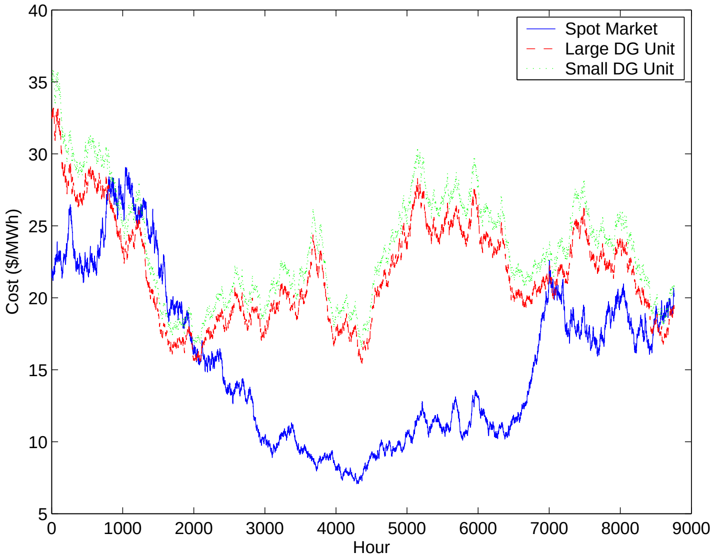

Figure 2: Simulated Electricity Price and On-Site Generation Costs

3.1 One-Unit DG System

We start by considering a microgrid that has installed only one DG unit.[5] In order to find the option value, F, of this investment over the test year, we first solve the SDP described in equations 1 through 6 and then subtract its minimised cost, V, from that of a microgrid that has no installed capacity, V^0, i.e., one that meets its entire load via spot purchases. A 95% confidence interval for the DG unit's option value may be constructed as follows:

$$\overline{F} \pm t_{N-1,0.975}\sqrt{\frac{\mathcal{S}^2(F)}{N}} \tag{12}$$

In equation 12, the sample mean, $\overline{F}$, is the difference between the mean minimum costs without and with DG installed, i.e., $\overline{F} = \overline{V^0} - \overline{V}$. Correspondingly, the sample variance of F is $\mathcal{S}^2(F) = \mathcal{S}^2(V^0) + \mathcal{S}^2(V) - 2Cov(V^0, V)$.

Using this procedure and equation 12, we calculate the implied option values of the single DG units under three conditions:

[5] This reduces the set of allowable states to either {E, (S,E)} or {E, (L,E)}. The SDP in equations 1 through 6 is modified accordingly.

- No start-up costs (NS): achieve cost minimisation by following a myopic policy of making the decision that minimises only immediate costs without considering future value functions, i.e., ignoring the second term in equation 1 when selecting from among the alternative choices.
- Start-up costs (SU): include start-up costs and proceed optimally using the LSMC simulation algorithm as outlined in equation 7 and Figure 1.
- Start-up costs with a myopic policy (SM): include start-up costs and use the myopic policy of case NS to make decisions.

Via these three cases, we illustrate that although start-up costs do not significantly reduce the option values of DG units, they must be taken into account by altering the operating policy to consider the tradeoff between current costs and future start-up costs. Indeed, we shall show that if a microgrid ignores this tradeoff and follows a myopic operating policy as in case MS, then the DG option value is statistically significantly reduced.

From Table 2, it can be seen first of all that the inclusion of start-up costs decreases the average option value of DG only slightly as long as the correct tradeoff is made between current cost minimisation and future start-up costs. In particular, there is a 3% reduction in the value of the 200 kW DG unit when start-up costs are imposed. For the 500 kW DG unit, the impact of start-up costs is slightly lower at 2.3% due to the larger unit's greater efficiency and low start-up costs relative to its generating cost. This decrease does not appear to be statistically significant at the 95% level as indicated by the overlapping confidence intervals for F between cases NS and SU in Table 3. On the other hand, if the microgrid does not adjust its operating policy to account for future start-up costs and continues to follow a myopic policy as in case NS when start-up costs are imposed, then the average option value of DG decreases by a much larger amount, i.e., 44% and 29% for the small and large DG units, respectively. As indicated in Table 3, this decrease is statistically significant at the 95% level as the confidence intervals for F between cases SU and SM are disjoint.

Case	200 kW DG	500 kW DG
NS	$\overline{F} = 3.67$, $\mathcal{S}(F) = 4.61$	$\overline{F} = 11.87$, $\mathcal{S}(F) = 13.16$
SU	$\overline{F} = 3.56$, $\mathcal{S}(F) = 4.61$	$\overline{F} = 11.60$, $\mathcal{S}(F) = 13.18$
SM	$\overline{F} = 2.07$, $\mathcal{S}(F) = 4.05$	$\overline{F} = 8.44$, $\mathcal{S}(F) = 12.68$

Table 2: Sample Mean and Standard Deviation for Individual DG Implied Option Values (in thousand $)

Case	200 kW DG	500 kW DG
NS	[3.38, 3.95]	[11.06, 12.69]
SU	[3.27, 3.84]	[10.79, 12.42]
SM	[1.81, 2.32]	[7.65, 9.22]

Table 3: Confidence Intervals for Individual DG Implied Option Values (in thousand $)

In order to determine the operational implications of start-up costs, we find start-up and shut-down price thresholds for the DG unit. We pick a representative hour during the test year to examine how the thresholds shift as a result of the start-up costs. Figure 3 is a scatterplot of the electricity prices and the DG generating costs at which the 200 kW DG unit is turned on (indicated by the blue circles) or turned off (indicated by the red crosses). Note that the region in which it is optimal to turn on the DG unit is disjoint from that in which it is optimal to turn off the DG unit. Indeed, the boundary between these two regions reflects the fact that a myopic policy is optimal, i.e., in the absence of start-up costs, it is optimal to start-up (shut-down) the DG unit immediately if the ratio of the DG generating cost to the electricity price falls below (exceeds) a critical value. For example, if the current DG generating cost is \$20/MWh and the DG unit is off, then the microgrid should wait until the electricity price is just above \$20/MWh before switching the unit on. Similarly, if the unit is on, then the microgrid should switch the unit off when the electricity price drops to \$20/MWh.

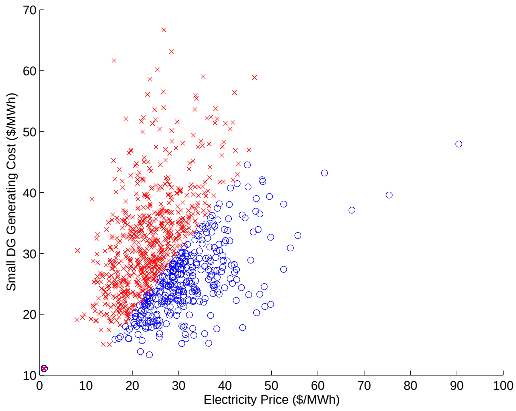

Figure 3: Operational Thresholds for 200 kW DG Unit (Case NS, Hour 4000)

As discussed previously, the presence of start-up costs implies that a myopic policy is no longer optimal since the microgrid needs to consider not only the current relative costs of meeting its load, but also the future costs, which may include start-up costs. Therefore, a similar scatterplot for case SU results in a zone of inaction between the two action regions (see Figure 4) in which it is optimal to wait and maintain the *status quo*, i.e., keep the DG unit on (off) if it is currently on (off). Effectively, for a given DG generating cost, the electricity price threshold at which to turn on (off) a DG unit that is currently off (on) is higher (lower) than in the NS case. For example, if the current DG generating cost is \$20/MWh and the DG unit is off (on), the microgrid waits until the electricity price is around \$23/MWh (\$17/MWh) before turning the unit on (off). The intuition for this hesitancy is that the microgrid wants to avoid a situation in which it turns on (off) a marginally cheaper (more expensive) DG unit only to have to turn it off (on) the following hour. In fact, both operational thresholds are affected because once a DG unit is on, the prospect of shutting it down only to have to turn it back on (and thus, pay the start-up cost) delays the shut-down decision. It is preferable to incur slightly higher energy costs in the current period by keeping a relatively expensive DG unit on than to pay start-up costs or face even higher costs in the future. Furthermore, the zone of inaction appears to widen as both the electricity price and DG generating cost increase, e.g., if the latter is \$40/MWh, then the electricity price needs to be nearly \$50/MWh (\$35/MWh) for an off (on) unit to be turned on (off). This increasing wedge between the two action zones could simply reflect a constant percentage by which the electricity price must exceed (fall below) the DG generating cost before action may be taken optimally. On the other hand, at higher prices, there may be more uncertainty about future prices, which then causes the microgrid to be more cautious in its decision making. In either case, although the implied option value of the DG unit is not significantly affected by start-up costs, their impact on the microgrid's operating policy is more profound.

We found that if start-up costs are present, but the microgrid continues to follow the myopic operating policy of case NS, then the implied option value of its DG unit is statistically significantly reduced. From the scatterplot in Figure 5, it now becomes clear why this is the case: following a myopic policy does not alter the "off" threshold relative to case NS, but shifts the "on" threshold much further to the right. This is because by ignoring future start-up costs, the microgrid readily turns off an active DG unit according

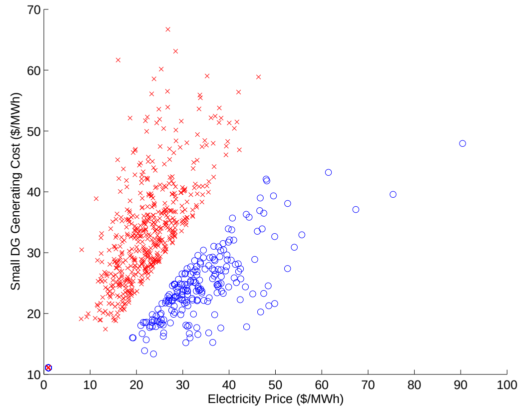

Figure 4: Operational Thresholds for 200 kW DG Unit (Case SU, Hour 4000)

to the threshold given in Figure 3, which subsequently puts it in a situation where it is not cost effective to use DG in the future unless the electricity price increases drastically so that the immediate energy cost savings outweigh the start-up costs of DG. In contrast, if it takes future start-up costs into account as in case SU, then it turns an active DG unit off only if the immediate cost savings outweigh *future* expected start-up costs. Hence, a microgrid that follows an optimal operating policy shifts its "off" and "on" thresholds slightly relative to case NS, whereas one that follows a myopic policy ignores not only future expected start-up costs in turning off an active DG unit (thereby maintaining the "off" boundary from case NS), but also future expected cost savings from turning on an idle DG unit (thereby causing the "on" boundary to shift more to the right than in case SU).

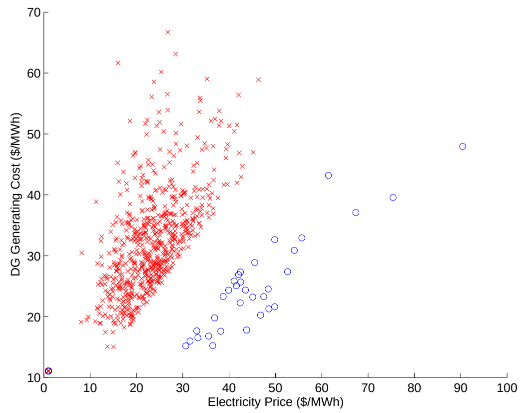

Figure 5: Operational Thresholds for 200 kW DG Unit (Case SM, Hour 4000)

The impact of start-up costs on the larger 500 kW DG unit is similar: a zone of inaction appears in case SU relative to case NS that results in a more hesitant microgrid (see Figures 6 and 7). Although it is not verified here statistically, this zone of inaction appears to be narrower with the 500 kW DG unit than with the 200 kW. In effect, the greater efficiency and relatively lower start-up cost of the larger DG unit imply that it is more flexible than the smaller one. Nevertheless, as indicated in Figure 8, if the large DG unit were operated in a myopic manner, then it, too, would lose a significant fraction of its option value as it would not trade off future expected cash flows with current ones. Hence, substantial alteration in the DG unit's operating policy is required in the presence of start-up costs even if these constraints have little significant impact on the microgrid's costs.

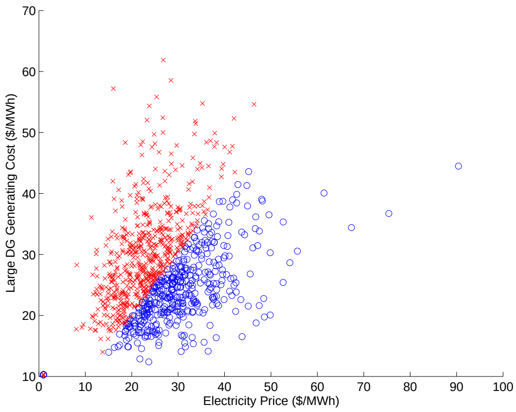

Figure 6: Operational Thresholds for 500 kW DG Unit (Case NS, Hour 4000)

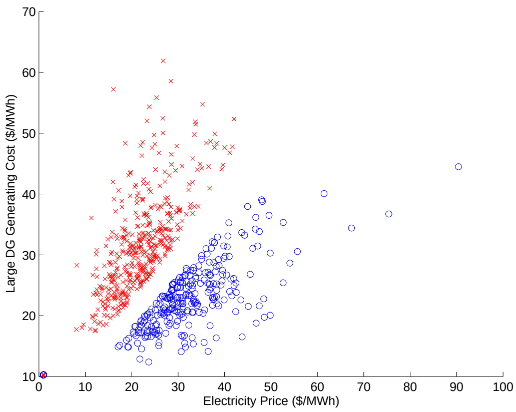

Figure 7: Operational Thresholds for 500 kW DG Unit (Case SU, Hour 4000)

3.2 Two-Unit DG System

By installing both DG units, the microgrid realises greater cost savings. However, the implied option value of the DG portfolio is no greater than the sum of the individual implied option values (see Table 4). Similar to the results from Section 3.1, we find that the presence of start-up costs does not statistically significantly alter the implied option value of the DG system, the average option value decreases by 2.5% in case SU. On the other hand, ignoring the presence of these seemingly insignificant costs can have profound implications for the system as the implied option value is reduced by 32% in the SM case relative to the NS case. As in Section 3.1, this decrease is statistically significant as the confidence intervals for F do not overlap.

In Figures 9 through 11, the impact of start-up costs on the DG system operating policy is indicated for a representative hour, with blue and light-green circles (red and cyan crosses) representing the the "on" ("off") thresholds for the small and large DG, respectively. Although we do not verify this formally,

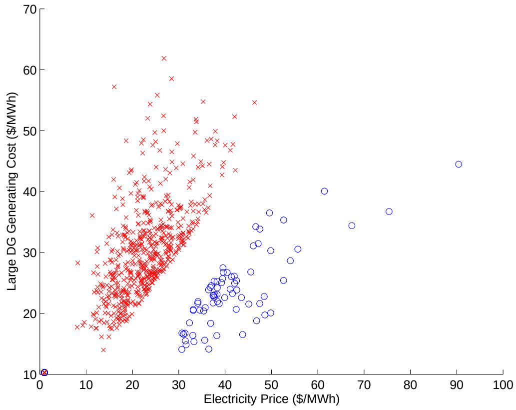

Figure 8: Operational Thresholds for 500 kW DG Unit (Case SM, Hour 4000)

Case	$\overline{F}$	$\mathcal{S}(F)$	95% Confidence Interval for F
NS	15.54	17.75	[14.44, 16.64]
SU	15.15	17.77	[14.05, 16.26]
SM	10.50	16.59	[9.47, 11.53]

Table 4: Sample Mean, Sample Standard Deviation, and 95% Confidence Intervals for Portfolio DG Implied Option Values (in thousand $)

it appears that with a two-unit DG system, the microgrid's operting policy in each of the three test cases is a simple superimposition of the single-unit operating policy. As in the single-unit case, the presence of start-up costs results in a zone of inaction, which, if neglected by following a myopic policy as in case MS, widens to affect the "on" threshold while leaving the "off" threshold unchanged from case NS. Effectively, the two DG units may be operated independently of each other because not only are the start-up costs independent, but also generating from one unit does not affect the microgrid's ability to use the other since the sum of the capacities is less than the microgrid's load.

4 Conclusion

The ongoing deregulation of electricity industries worldwide provides opportunities for microgrids to evolve according to the needs of end-use consumers by incorporating DG and CHP applications where beneficial. In this paper, we make a stylised attempt to examine the impact of modest start-up costs on DG value and operation within a stochastic setting. By taking a real options approach, we find that although the impact of start-up costs on the implied option value of DG is minor, their operational implications are certainly more profound. In effect, the presence of start-up costs forces the microgrid to trade off current cash flows with estimates of future expected cash flows before making any operational decisions since future cash flows are affected by current actions and states. By contrast, without start-up costs, the microgrid may proceed to make decisions in a myopic manner, i.e., consider only current cash flows and states in making its decision because future cash flows are independent of current actions and states. Factoring this dependency into the microgrid's decisions causes there to be a zone of inaction between the "on" and "off" thresholds for DG as it becomes preferable to wait before the electricity price (DG generating cost) more than exceeds the DG generating cost (electricity price) before turning the DG unit on (off). Indeed, this hesitancy results from the fact that the microgrid must now include

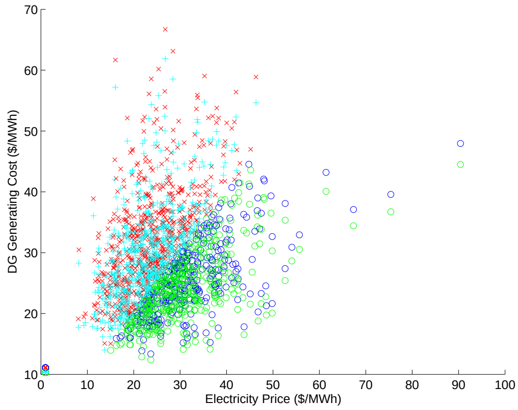

Figure 9: Operational Thresholds for System of 200 kW and 500 kW DG Units (Case NS, Hour 4000)

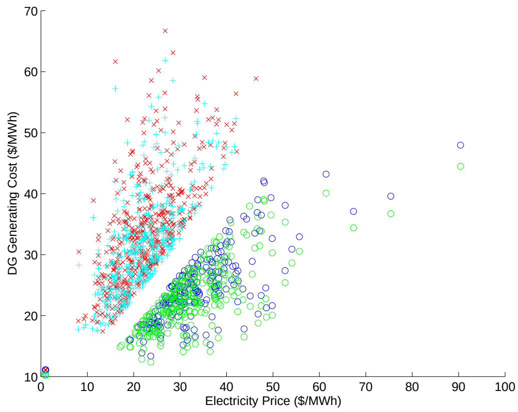

Figure 10: Operational Thresholds for System of 200 kW and 500 kW DG Units (Case SU, Hour 4000)

future expected start-up costs as implicit opportunity costs of turning on an idle DG unit. Since the additional cost makes switching to the "on" state less attractive, the microgrid maintains the *status quo* over an intermediate range of prices. Similarly, the presence of start-up costs causes the microgrid to postpone turning off an active DG unit because future expected start-up costs are subtracted from the current cost savings of using electricity purchases, thereby leading to inaction as long as the electricity price is not relatively low. If such a tradeoff is ignored, i.e., the microgrid proceeds myopically in the presence of start-up costs, then the zone of inaction widens, resulting in significantly higher costs. In particular, the "off" threshold is not affected since future expected start-up costs are ignored. However, the "on" threshold is shifted far to the right as the subtraction of current start-up costs from current cost savings of using DG without accounting for future expected cost savings from an "on" DG unit reduces the benefits of DG. Therefore, from our model, a microgrid manager can infer not only the option values of its DG, but also identify optimal operating schedules.

In order to focus on such operational implications, we have neglected many real-world features in our stylised model. For instance, we do not consider the transmission and distribution costs of electricity purchases, which would make DG units more valuable than is implied in this work. Our analysis could also benefit from more rigorous treatment of DG O&M costs as we allocate only incremental variable O&M costs as start-up costs. Omission of CHP applications also understates the value of DG. This

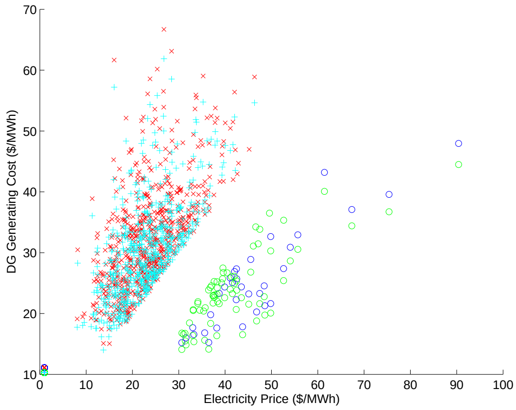

Figure 11: Operational Thresholds for System of 200 kW and 500 kW DG Units (Case SM, Hour 4000)

feature may also affect DG operating schedules depending on the degree to which electric and heat loads are coincident. We propose to incorporate CHP as an option to upgrade in future work. In this context, the investment decision should also be modelled, although it is more common for such decisions to be driven by long-term price factors rather than the short-term ones we address here (see [11]). Finally, from a modelling perspective, since we examine a relatively short time horizon, we should incorporate seasonality, daily peak prices, and spikes in both price processes.

References

[1] Abu-Sharkh, A, RJ Arnold, J Kohler, R Li, T Markvart, JN Ross, K Steemers, P Wilson, and R Yao (2006), "Can Microgrids Make a Major Contribution to UK Energy Supply?," *Renewable and Sustainable Energy Reviews*, 10(2): 78–127.

[2] Boyle, PP (1977), "Options: A Monte Carlo Approach," *Journal of Financial Economics*, 4(3): 323–338.

[3] Boyle, PP, J Evnine, and S Gibbs (1989), "Numerical Evaluation of Multivariate Contingent Claims," *The Review of Financial Studies*, 2(2): 241–250.

[4] CERTS Berkeley Symposium on Microgrids (2005), UC Berkeley Faculty Club, UC Berkeley, Berkeley, CA, USA, 17 June 2005 (presentations available at `http://der.lbl.gov/CERTSmicrogrids.html`).

[5] Cox, JC, SA Ross, and M Rubinstein (1979), "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, 7(3): 229–263.

[6] Deng, S-J, B Johnson, and A Sogomonian (2001), "Exotic Electricity Options and the Valuation of Electricity Generation and Transmission Assets," *Decision Support Systems*, 30(3): 383–392.

[7] Deng, S-J and SS Oren (2003), "Incorporating Operational Characteristics and Start-Up Costs in Option-Based Valuation of Power Generation Capacity," *Probability in the Engineering and Informational Sciences*, 17(2): 155–181.

[8] Gumerman, E, R Bharvirkar, K LaCommare, and C Marnay (2003), "Evaluation Framework and Tools for Distributed Energy Resources," Ernest Orlando Lawrence Berkeley

National Laboratory Technical Report LBNL-52079, Berkeley, CA, USA (available at `http://eetd.lbl.gov/ea/emp/der-pubs.html`).

[9] Longstaff, FA and ES Schwartz (2001), "Valuing American Options by Simulation: A Simple Least-Squares Approach," *The Review of Financial Studies*, 14(1): 113–147.

[10] Marnay, C and G Venkataramanan (2006), "Microgrids in the Evolving Electricity Generation and Delivery Infrastructure," in Proceedings of the 2006 IEEE Power Engineering Society General Meeting, Montréal, Québec, Canada (18–22 June 2006).

[11] Näsäkkälä, E and Fleten, S-E (2005), "Flexibility and Technology Choice in Gas Fired Power Plant Investments," *Review of Financial Economics*, 14(3-4): 371–393.

[12] Pepermans, G, J Driesen, D Haeseldonckx, R Belmans, and W D'haeseleer (2005), "Distributed Generation: Definition, Benefits and Issues," *Energy Policy*, 33(6): 787–798.

[13] Rendleman, RJ and BJ Bartter (1979), "Two-State Option Pricing," *Journal of Finance*, 34(5): 1093–1110.

[14] Siddiqui, AS, C Marnay, O Bailey, and K LaCommare (2005), "Optimal Selection of On-Site Power Generation with Combined Heat and Power Applications," *International Journal of Distributed Energy Resources*, 1(1): 33–62.

[15] Siddiqui, AS, C Marnay, JL Edwards, R Firestone, S Ghosh, and M Stadler (2005), "Effects of Carbon Tax on Microgrid Combined Heat and Power Adoption," *Journal of Energy Engineering*, 131(1): 2–25.

[16] Siddiqui, AS, C Marnay, R Firestone, and N Zhou (2005), "Distributed Generation with Heat Recovery and Storage," in 7th European Conference Proceedings of the International Association for Energy Economics, Bergen, Norway (28–30 August 2005).

[17] Tseng, C-L and G Barz (2002), "Short-Term Generation Valuation: A Real Options Approach," *Operations Research*, 50(2): 297–310.

[18] Zhao, T, SK Sundararajan, and C-L Tseng (2004), "Highway Development Decision-Making Under Uncertainty: A Real Options Approach," *Journal of Infrastructure Systems*, 10(1): 23–32.