Phase space structures generated by an absorbing obstacle in a streaming plasma

P. Guio
Institute for Theoretical Astrophysics, University of Oslo, Norway

H. L. Pécseli
Physics Department, University of Oslo, Norway

Received 20 August 2003; revised 29 December 2003; accepted 30 December 2003; published 11 February 2004.

[1] The dynamic behavior of an ion flow around an obstacle in a collisionless plasma is investigated. The obstacle consists here of an absorbing cylinder, and a 2 dimensional electrostatic particle-in-cell simulation is used to study the flow characteristics. The formation of irregular filamented density depletions, oblique to the flow, is observed. The dynamics of these structures depend on the physical parameters of the plasma. The structures form at the edges of the wake behind the obstacle, in a region with a strong velocity shear, and are found to be associated with phase-space vortices, observed specially in the velocity direction perpendicular to the flow. The results can be of interest in the interpretation of structures in space plasmas as observed by instrumented space crafts. *INDEX TERMS:* 2471 Ionosphere: Plasma waves and instabilities; 2483 Ionosphere: Wave/particle interactions; 2753 Magnetospheric Physics: Numerical modeling. **Citation:** Guio, P., and H. L. Pécseli (2004), Phase space structures generated by an absorbing obstacle in a streaming plasma, *Geophys. Res. Lett.*, *31*, L03806, doi:10.1029/2003GL018461.

1. Introduction

[2] The problem of a plasma flowing past an obstacle has been studied for different system sizes, as well as for different plasma parameter regimes. For example [*Stangeby and Allen*, 1971] studied analytically the stationary flow pattern of a compressible, supersonic, irrotational and inviscid flow of drifting cold ions around a cylindrical obstacle. Considering different and much larger length scales, [*Farrell et al.*, 1998; *Birch and Chapman*, 2002] studied the kinetic effects of solar wind electrons and the IMF direction in the wake formation and evolution past the Moon. Filling of the void behind an obstacle moving rapidly through a fully ionized plasma (the shuttle Orbiter, for instance) were studied by [*Singh et al.*, 1989] by a combination of numerical and analytical methods.

[3] In the present study, we consider absorbing cylindrical obstacles with diameters on the Debye length scale, immersed in a plasma consisting of an ion species having a subsonic drift velocity relative to the obstacle. The problem is analyzed here by numerical methods.

2. Numerical Simulations

[4] We use the same particle-in-cell simulator as [*Guio et al.*, 2003], with the extension that it now allows for inclusion of absorbing surfaces. The code is parallelized and allows for simulations with a large number of particles over wide rectangular regions.

[5] We assume the electrons to be isothermally Boltzmann distributed at T_e for all times, leading to a nonlinear Poisson equation, $\nabla^2\phi = (e/\epsilon_0)(n_0\exp(e\phi/T_e) - n)$ where n is the ion density, assuming singly charged ions. Because of the assumed isothermal Boltzmann distribution of the electron component, the code does not account for electron kinetic phenomena due to, for instance, distorted electron velocity distribution functions, or electron current instabilities. On the other hand, the code offers a great advantage in needing attention only to the time scale of the ion dynamics. We have typically 3×10^6 particles in a simulation.

[6] A $10\lambda_i$ diameter absorbing circular obstacle is placed at the origin of the simulation region. Every ion crossing the surface of the obstacle is removed from the simulation, and the net charge density inside is fixed to zero. This solution allows for a simple treatment of the obstacle, which is here assumed to be at a constant potential, equal to the plasma potential. The ions have a net flow along the y-direction, as ensured by injection of particles at the boundaries, with the number of injected particles being calculated from the flux of a drifting Maxwellian distribution.

3. Results

[7] If we choose the ion flow velocity to be supersonic, we obtain the well known sound-wave cones [*Guio and Pécseli*, 2003]. One feature of the present analysis is that only moderate flow velocities V_0 are used, below the sound speed C_s, but larger than the ion thermal velocity, u_{ti}. Figure 1 shows the two-dimensional electrostatic potential at 6 different times, measured in units of the ion plasma period. For this particular simulation we have $T_e/T_{i0} = 20$, but the results are representative for other, smaller, temperature ratios as well. To simulate conditions in nature, where the ion temperature is often largest in the direction of the ion flow, we used $T_{i\|} \equiv T_{i0} = 5T_{i\perp}$ for the ion population injected at the boundary. In preparing the input data for the present simulations, we selected parameters to give a clear separation between C_s and u_{ti}. Here, we used $V_0 = 0.25\ C_s$.

[8] We observe the formation of long potential filaments behind the obstacle. In order to visualize the time variation of the potential we place a dense probe array positioned in the x-direction (perpendicular to the flow velocity) at a downstream position of $40\lambda_i$. The time evolution is sampled for 400 ion plasma periods. Results are shown in Figure 2.

0094-8276/04/2003GL018461

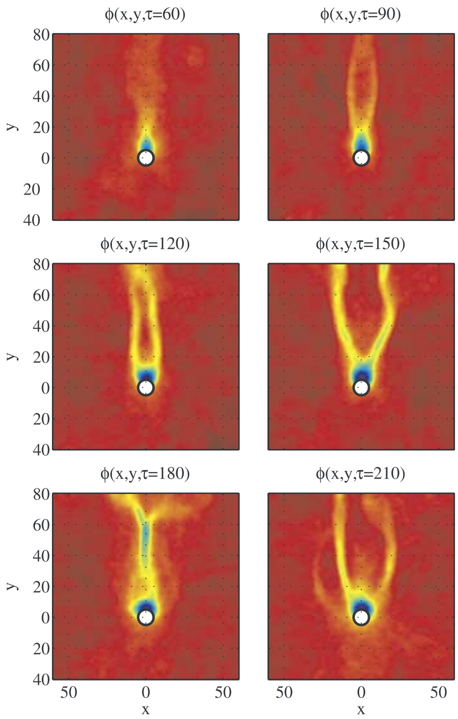

Figure 1. Two-dimensional electrostatic potential at 6 different times of the simulation. The color scale has red as maximum, where $e\phi/T_i \approx 0.2$ and deep blue as minimum, where $e\phi/T_i \approx -2.5$, right behind the obstacle.

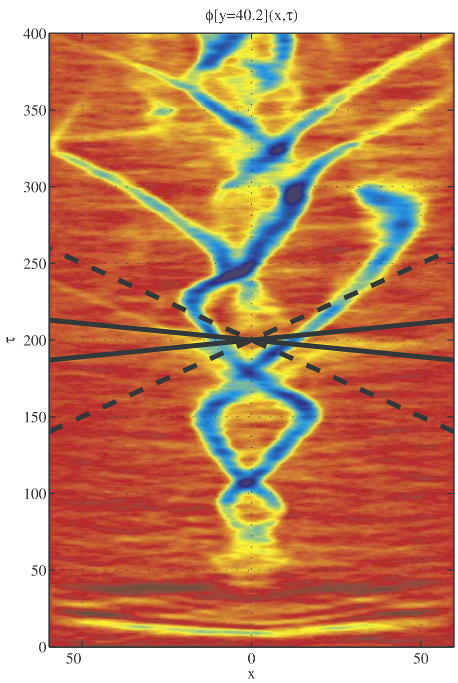

Figure 2. Time variation of the electric potential detected at a probe array placed at a downstream position of $40\lambda_i$. The time evolution is sampled with the numerical time resolution for 400 ion plasma periods. As a reference, we inserted the slopes of the ion sound velocity (solid line) and the ion thermal velocity (dashed line), in the middle of the figure. The color scale has red as maximum, where $e\phi/T_i \approx 0.2$ and deep blue as minimum, where $e\phi/T_i \approx -1.4$.

The formation time of the structures is approximately 100 ion plasma periods, and their saturated widths approximately $10\lambda_i$. We note a rather irregular motion of the filaments. At several instants we observe a coalescence of two vortices, e.g., at $t \approx 105$ and $t \approx 180$. The local slope of the structures in the $\{x, t\}$-diagram gives the local velocity component in the x-direction, which is here the dominant one. Disruptions, and abrupt changes of the structures are often associated with radiation of small amplitude slow waves: an example is noticeable for instance in Figure 2, around $(x, t) \approx (0, 180)$. The x-component of the velocity of these fluctuations is found to be between the ion thermal velocity and the speed of sound. If we reduce T_e/T_i, filaments still form, but they become thinner and more irregular. While the present parameters typically give two filaments as in Figure 1, a reduced temperature ratio can give a larger number. In all cases they start out from the wake immediately behind the obstacle.

[9] In order to obtain a better understanding of the nature of the observed structures, we analyze the phase space spanned by the $\{x, v_x\}$-coordinates, at selected times. Results are shown in Figure 3. The conspicuous features of the potential structures are associated with phase space vortices. These have been observed experimentally [*Pécseli et al.*, 1984], as well as in a number of numerical simulations [*Sakanaka*, 1972; *Børve et al.*, 2001; *Daldorff et al.*, 2001; *Guio et al.*, 2003]. In particular, the coalescence observed in Figure 2 is associated with a phase space coalescence, as found in Figure 3 at for instance $t = 180$. The velocities of these structures seems to be at or below the ion thermal velocity, consistent with analytical results, which demonstrate that these Bernstein-Greene-Kruskal (BGK) equilibria [*Bernstein et al.*, 1957] can have in principle any characteristic velocity, as long as they are embedded into the velocity distribution function. The radiation of small amplitude fluctuations is here seemingly associated with changes in ion phase space. It is interesting to note that also [*Singh et al.*, 2001] observe radiation of

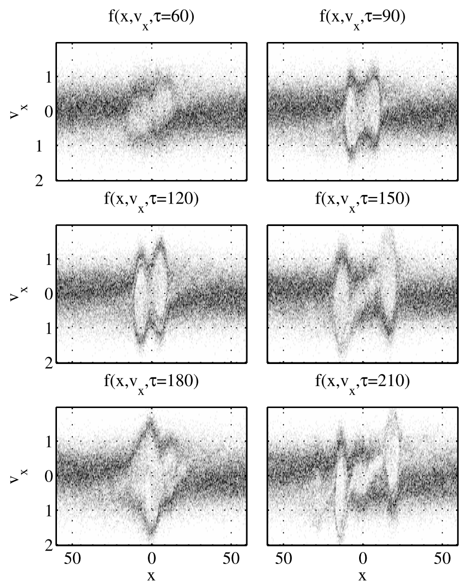

Figure 3. A section of phase space, spanned by $\{x, v_x\}$, shown at selected times.

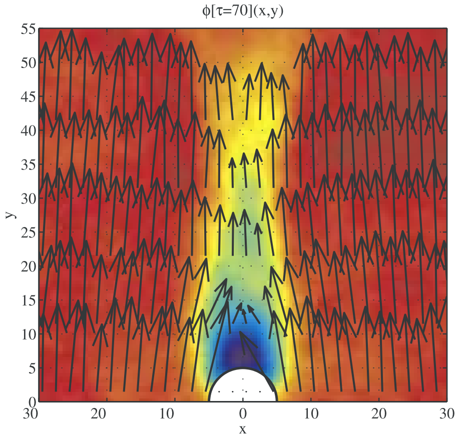

Figure 4. Diagram for the plasma flow velocity. For simplicity, we show only a restricted domain of the simulation space, see Figure 1 for color code.

small amplitude waves when phase space vortices undergo rapid changes (in their case electron holes radiating plasma waves). Related observations were made by [*Newman et al.*, 2001] and [*Oppenheim et al.*, 2001].

4. Discussions

[10] Many observations from plasmas in the Earth's near environment has revealed the presence of seemingly long lived structures. Numerical and experimental results have demonstrated that vortical structures in phase space have the observed characteristics. In simulations with full electron as well as ion dynamics [*Omura et al.*, 2003], phase space structures were observed in ion as well as electron phase space (best seen in electron phase space), as a result of an applied electric field which drives a current along magnetic field lines. These results are expected to be relevant for the interpretation of data from the GEOTAIL spacecraft. Those simulations indicate that the origin of the phase space structures is a kinetic current driven instability. A combined analytical-numerical study of ion-ion beam instabilities [*Pécseli and Trulsen*, 1982] demonstrated that also in this case ion phase space structures were formed as a result of a linear kinetic instability.

[11] Two possible, but basically different, instability mechanisms responsible for the formation of the elongated perturbations can be identified in our simulations: one is a fluid type instability associated with the velocity shear that exists at the edges of the shadow region. The other one is a kinetic instability, generated by the oppositely directed ion beams that flow into the wake from the two sides of the shadow region.

[12] For the velocity shear instability, we are dealing with a compressible flow, rather than the "classical" incompressible case [*Chandrasekhar*, 1961]. The theoretical stability analysis for the present case is rather lengthy, and is not reproduced here. It is found that perturbations propagating *strictly* along the velocity gradient are stable, i.e., a temporal growth requires perturbations to be at an angle to the velocity gradient.

[13] In Figure 4, we demonstrate that a localized velocity shear is present in the flow. The average, or fluid, velocity is here obtained numerically by integrating the velocity distribution to give

$$\mathbf{V}(\mathbf{r}, t) \equiv \int\int \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) dv_x dv_y \Bigg/ \int\int f(\mathbf{r}, \mathbf{v}, t) dv_x dv_y,$$

given by small arrows, while the color coding gives the potential variation. As expected, the shear is found at the edge of the obstacle: in the shadow of the cylinder we have the plasma being moving slowly, while it flows unimpeded past the object outside the shadow region. We see clearly a "channel" with small flow velocity behind the obstacle, with larger velocity vectors in the surrounding flow. The region of pronounced velocity shear is the one where the potential structures develop, see Figures 1 and 2. In presenting Figure 4, we selected $t = 70$, i.e., a time where the potential structures begin to develop, see Figure 1.

[14] We expect the lifetime of the structures to be infinite, but their length in the ion streamwise direction to be finite, since they are constantly maintained by the velocity shear right behind the obstacle. It is not the same particles which constitute a structure at all times. Studies of the lifetime of ion phase space vortices [*Pécseli et al.*, 1984; *Børve et al.*, 2001] refer to initial value problems, and do not apply for the present case.

[15] The kinetic instability mentioned before is similar to the one studied by, for instance, [*Singh et al.*, 1986, 1989; *Samir et al.*, 1989], although those investigations were primarily concerned with objects moving at super-sonic speeds. Basically, the ion beams enter from both sides of the shadow region, and when they mix, a two ion stream region may develop. For the present condition, where the flow is sub-sonic, a nontrivial part of the tail population of the Maxwellian ion distribution can flow into the shadow region, and as far as the $\{x, v_x\}$-part ion phase space is concerned, it contributes with a particle population at zero average velocity. We can estimate the x-component of the ion beam velocity by noting that potential drops at the edges of the shadow region are $\Delta\phi \approx T_i/e$, for the case analyzed here, see Figure 4. Consequently, the v_x-component of the average ion velocity is here of the order of the ion thermal velocity in the wake region. In a homogeneous plasma, this velocity is insufficient to give rise to an ion-ion beam instability [*Fried and Wong*, 1966], but in the present shear flow it might be that the two instabilities enhance each other. Our preference for the shear instability in the interpretation of the present simulations is based on the observation that the structures seem to develop at the location of the maximum shear, and elongated in a direction almost normal to the gradient in plasma velocity. It is, however, evident that the relative importance of the two instabilities may change, depending on plasma parameters. If we, for instance, increase V_0, we can clearly identify the two counter-streaming ion beams in the transverse direction of the wake.

[16] We have investigated the present problem by a fully three dimensional code as well. That analysis is significantly more time consuming, but gives essentially the same results. Our observations are therefore not restricted by the dimensionality of the code. The formation of phase space structures seems to require $T_e > T_i$, which is in agreement with numerical studies by [*Pécseli et al.*, 1984]. An ion temperature anisotropy seems to be of minor importance. The main feature seems to be that isotropic ion temperatures allow

wider excursions of structures in their motion in the direction perpendicular to the flow velocity. Larger obstacles tend to give rise to the formation of somewhat wider ("fatter") structures. For a diameter of $1.5\lambda_i$, or less, of the obstacle, it is no longer possible to observe any structures, but for a diameter of $2.5\lambda_i$, or larger, they are clearly discernible.

5. Conclusion

[17] In the present communication, we studied plasma flows past an absorbing obstacle, with particular attention to subsonic flow velocities. We demonstrated that the wake behind the obstacle is associated with a reduced plasma drift, as to be expected, while we have a larger flow velocity in the surrounding plasma. The plasma in the wake is unstable, and breaks up into elongated structures, which in the saturated stage are associated with phase space vortices.

[18] One somewhat unpleasant observation based on our studies can be that obstacles in a plasma flow (or obstacles moving with moderate velocities through a stationary plasma) can generate phase space structures. If we associate an obstacle as the one investigated in the present study with a rocket or a satellite, or a part of one, we might anticipate that the filamentary structures can be observed by probes on the same space craft, and incorrectly be associated with natural phenomena occurring in the ambient plasma. This scenario deserves, in our opinion, some attention.

[19] A high altitude satellite like the GEOTAIL spacecraft is small in comparison to the local electron Debye length (with $n \approx 0.1 \times 10^6$ m^{-3}, $T_e \approx 100$ eV). The FAST satellite is comparable to or larger than the Debye length (with $n \approx 6 \times 10^6$ m^{-3}, $T_e \approx 700$ eV). These two satellites are close to being ineffective "point-sources", in the present context. However, a lower altitude satellite like FREJA is approximately $30 \times 6\ \lambda_{De}$, in terms of local plasma parameters. With a typical sound speed of $(7.4 \pm 15\%) \times 10^6$ m s^{-1} (taken from standard tables) for a hydrogen plasma, and a satellite speed of 7.5×10^6 m s^{-1}, we have the FREJA satellite propagating with Mach numbers varying around unity. Indeed, observations of sporadic localized structures have been reported [*Dovner et al.*, 1994], in addition to other phenomena (such as lower-hybrid cavities etc.), which are not discussed here. We also expect our results to apply for sounding rockets probing the collisionless plasma in the ionospheric E-region. To discuss the wake of the shuttle [*Singh et al.*, 1989] we will have to consider supersonic flows, but the basic problem is relevant also there.

[20] **Acknowledgments.** This work was in part supported by the Norwegian Research Council's grant 146467/420, "Solar and Heliospheric Plasmas".

References

Bernstein, I. B., J. M. Greene, and M. D. Kruskal (1957), Exact nonlinear plasma oscillations, *Phys. Rev.*, *108*, 546–550.
Birch, P. C., and S. C. Chapman (2002), Two dimensional particle-in-cell simulations of the lunar wake, *Phys. Plasmas*, *9*, 1785–1789.
Børve, S., H. L. Pécseli, and J. Trulsen (2001), Ion phase-space vortices in 2.5-dimensional simulations, *J. Plasma Phys.*, *65*, 107–129.
Chandrasekhar, S. (1961), *Hydrodynamic and Hydromagnetic Stability*, Oxford Univ. Press, London.
Daldorff, L. K. S., P. Guio, S. Børve, H. L. Pécseli, and J. Trulsen (2001), Ion phase space vortices in 3 spatial dimensions, *Europhys. Lett.*, *54*, 161–167.
Dovner, P. O., A. I. Eriksson, R. Boström, and B. Holback (1994), Freja multiprobe observations of electrostatic solitary structures, *Geophys. Res. Lett.*, *21*(17), 1827–1830.
Farrell, W. M., K. M. L., and J. T. Steinberg (1998), A simple simulation of a plasma void: Applications to Wind observations of the lunar wake, *J. Geophys. Res.*, *103*, 23,653–23,660.
Fried, B. D., and A. Y. Wong (1966), Stability limits for longitudinal waves in ion beam-plasma interaction, *Phys. Fluids*, *9*, 1084–1089.
Guio, P., and H. L. Pécseli (2003), Radiation of sound from a charged dust particle moving at high velocity, *Plasma Phys.*, *10*, 2667–2676.
Guio, P., S. Børve, L. K. S. Daldorff, J. P. L. P. Michelsen, H. L. Pécseli, J. J. Rasmussen, K. Saeki, and J. Trulsen (2003), Phase space vortices in collisionless plasmas, *Nonlin. Processes Geophys.*, *10*, 75–86.
Newman, D. L., M. V. Goldman, M. Spector, and F. Perez (2001), Dynamics and instability of electron phase-space tubes, *Phys. Rev. Lett.*, *86*, 1239–1242.
Omura, Y., W. J. Heikkila, T. Umeda, K. Ninomiya, and H. Matsumoto (2003), Particle simulations of plasma response to an applied electric field parallel to magnetic field lines, *J. Geophys. Res.*, *108*(A5), 1197, doi:10.1029/2002JA009573.
Oppenheim, M., G. Vetoulis, D. L. Newman, and M. V. Goldman (2001), Evolution of electron phase-space holes in 3d, *Geophys. Res. Lett.*, *28*(9), 1891–1894.
Pécseli, H. L., and J. Trulsen (1982), Nonlinear evolution of the ion-ion beam instability, *Phys. Rev. Lett.*, *48*, 1355–1358.
Pécseli, H. L., J. Trulsen, and R. Armstrong (1984), Formation of ion phase-space vortexes, *Phys. Scripta*, *29*, 241–253.
Sakanaka, P. H. (1972), Beam-generated collisionless ion-acoustic shocks, *Phys. Fluids*, *15*, 1323–1327.
Samir, U., R. H. Comfort, N. Singh, K. S. Hwang, and N. H. Stone (1989), Insight into theory-experiment comparisons of wake measurements in the plasmasphere, *Planet. Space. Sci.*, *37*, 873–880.
Singh, N., H. Thiemann, and R. W. Schunk (1986), Studies of counter-streaming plasma expansion, *Phys. Scripta*, *33*, 355–369.
Singh, N., K. H. Wright Jr., N. H. Stone, U. Samir, and K. S. Hwang (1989), On the interpretation of measured ion streams in the wake of the shuttle orbiter in terms of plasma expansion processes, *J. Geophys. Res.*, *94*(A9), 12,075–12,080.
Singh, N., S. M. Loo, and E. Wells (2001), Electron hole as an antenna radiating plasma waves, *Geophys. Res. Lett.*, *28*(7), 1371–1374.
Stangeby, P. C., and J. E. Allen (1971), Transonic plasma flow past an obstacle, *J. Plasma Phys.*, *6*, 19–32.

P. Guio, University of Oslo, Institute for Theoretical Astrophysics, Blindern, N-0315 Oslo, Norway. (patrick.guio@astro.uio.no)
H. L. Pécseli, University of Oslo, Physics Department, Blindern, N-0316 Oslo, Norway. (hans.pecseli@fys.uio.no)