Effect of Nonlinearity on Adiabatic Evolution of Light

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We investigate the effect of nonlinearity in a system described by an adiabatically evolving Hamiltonian. Experiments are conducted in a three-core waveguide structure that is adiabatically varying with distance, in analogy to the stimulated Raman adiabatic passage process in atomic physics. In the linear regime, the system exhibits an adiabatic power transfer between two waveguides which are not directly coupled, with negligible power recorded in the intermediate coupling waveguide. In the presence of nonlinearity the adiabatic light passage is found to critically depend on the excitation power. We show how this effect is related to the destruction of the dark state formed in this configuration.

The adiabatic theorem describes one of the most powerful concepts in quantum physics [1]. It states that if the parameters of a quantum system evolve slowly enough in time, the associated initial eigenstates will be preserved, and there will be no exchange of energy between them. This well-studied theorem finds wide applications in diverse areas of science ranging from molecular physics to quantum field theory, from chemistry to nuclear physics. A close reexamination of the adiabatic principles led to the discovery of Berry's geometric phase [2]—known to occur ubiquitously in many processes in nature [3]. Quite recently, quantum adiabatic methods were suggested as a basis for a new class of algorithms meant to address NP (nondeterministic polynomial time)-complete problems within the framework of quantum computing [4]. In addition, techniques exploiting an adiabatic passage provide practical approaches in achieving nearly complete population transfer between two quantum states [5–8]. One such example of coherent adiabatic excitation is stimulated Raman adiabatic passage (STIRAP) that makes use of two appropriately prepared laser pulses in order to couple two nondegenerate metastable states via an intermediate level, without any appreciable excitation of the intermediate state [5,6,9].

One of the underlying—and sometimes limiting—assumptions of the adiabatic theorem is the presumed intrinsic linearity of the system, a condition that is often not met under actual experimental conditions. For example, nonlinearity comes into play in various adiabatically evolving systems such as Bose-Einstein condensates in slowly varying potentials or fields [10–13] and nonlinear optical processes [14,15]. Of course, the question naturally arises on how nonlinear effects may influence such adiabatic transfer processes [11–13,16,17]—an aspect that has so far eluded experimental observation.

In this Letter we consider the influence of nonlinearity in systems described by an adiabatically evolving Hamiltonian. Experiments are conducted in a system of coupled optical waveguides in which the distance between channels changes slowly along the propagation axis. Nonlinear optical waveguides, described by the nonlinear Schrödinger equation, allow one to take a simple and direct experimental look at the interplay between adiabatic evolution and nonlinearity. In addition they provide a direct analogy with various other quantum processes. These include time-dependent quantum effects in atomic physics, Bose-Einstein condensates in time varying traps, and time-dependent quantum-well potentials—all described in different regimes by the same evolution equations presented here. As an example, we use a three-waveguide structure that reproduces the STIRAP process in atomic physics [18]. In the linear regime, the system exhibits a complete and irreversible power transfer between two waveguides that are not directly coupled, via an intermediate channel. Remarkably, this intermediate waveguide carries no significant field amplitude during the power exchange. In the nonlinear regime, the adiabatic light passage is found to critically depend on the excitation power levels. We show how this effect is related to the destruction of a dark state formed in the STIRAP configuration [12].

Consider a system of three single-mode, evanescently coupled nonlinear waveguides [denoted as 1, 2, and 3, see Fig. 1(a)]. The waveguides are identical in shape and have a constant width along the propagation direction, z. However, the distances between the waveguides vary along the propagation. Waveguides 1 and 3 are parallel to each other, while waveguide 2 is oblique; it is closer to waveguide 1 at $z = 0$, and closer to waveguide 3 at $z = L$, where L is the sample length [see Fig. 1(a)]. As a consequence, the coupling rates between the waveguides vary
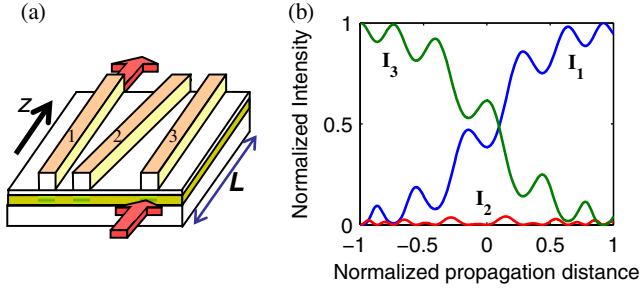
(a)   (b)

FIG. 1 (color online).   (a) A schematic view of the STIRAP sample. The relative distance between the coupled waveguides (denoted 1, 2, and 3) changes slowly along the propagation axis z, resulting in slowly changing couplings rates between the waveguides. (b) Adiabatic light passage as calculated from Eq. (2) for a 3-core structure with $\alpha = 66$ m^{-1}, $L = 3$ cm, $\kappa = 600$ m^{-1} (see text for definitions). The intensity in every channel is plotted as a function of normalized distance.

slowly along the propagation. At $z = 0$ the coupling between waveguides 1 and 2 (C_{12}) is strong, while the coupling between waveguide 2 and 3 (C_{23}) is weak. At the output of the system (at $z = L$) the situation is reversed, i.e., $C_{23} > C_{12}$. The coupling between waveguides 1 and 3 is practically zero in this configuration.

The evolution of the modal amplitudes in these three waveguides can be described by the following set of coupled discrete nonlinear Schrödinger equations:

$$i\frac{\partial E_n}{\partial z} + \beta_n E_n + \sum_m C_{n,m}(z)E_m + \Gamma |E_n|^2 E_n = 0 \quad (1)$$

where $n = 1, 2, 3$, E_n is the wave amplitude in waveguide n, β_n is the longitudinal wave vector (propagation constant) for the mode in waveguide n, and the summation is carried out on nearest neighbors. The last term in Eq. (1) accounts for the nonlinear dependence of the on-site wave vector β, where Γ is associated with the Kerr nonlinear coefficient of the waveguide structure. This term can be neglected at low light power levels.

In the linear limit, the description of this system by Eq. (1) carries a perfect analogy to the STIRAP process first described in the framework of atomic physics [5,6]. This surprising process leads to complete population transfer between two atomic levels for which a direct transition is forbidden, via a third level. However, in the adiabatic limit the intermediate level is never populated during the process [6]. Indeed, the equations used to describe the STIRAP effect in atomic physics are identical to Eq. (1) in the linear limit. In this analogy z replaces time, the amplitude in each waveguide corresponds to the amplitude in each atomic level, and the waveguide coupling plays the role of the Rabbi coupling of the atomic levels caused by resonant electromagnetic radiation. Identical values of the parameter β for coupled waveguides represent zero detuning of the electromagnetic radiation from the level spacing. A linear STIRAP scheme was recently suggested in an optical system using a different analogy that required an

imprint of periodic gratings or bending of the structure along the propagation axis, to introduce coupling between dissimilar waveguides [19,20]. An implementation using a simple geometry similar to the one discussed here was proposed by Paspalakis [21], and recently implemented in the linear regime by Longhi and co-workers [18].

To theoretically analyze the linear response ($\gamma = 0$) of the system shown in Fig. 1(a) we recall that the coupling coefficient between two evanescently coupled waveguides varies exponentially with the separation distance [22]. As a result, for a structure of length L, the two coupling constants are found to vary according to $C_{12}(z) = \kappa \exp[-\alpha(z - L/2)]$ and $C_{23}(z) = \kappa \exp[\alpha(z - L/2)]$, where κ is the coupling strength at $z = L/2$ and α is a slow adiabatic parameter related to the slope of waveguide 2, that is $\gamma \equiv \alpha/\kappa \ll 1$. If at the input of this system the third waveguide is excited, i.e., $E_3(0) = 1$, then by employing WKB expansion methods one can show that to a very good approximation the field in the first waveguide evolves according to

$$E_1(z)e^{-i\beta z} = \frac{A\sqrt{1 + e^{4t_0}}}{\sqrt{1 + e^{-4t}}} - \frac{A\sqrt{1 + e^{-4t_0}}}{\cos\phi\sqrt{1 + e^{4t}}}\cos[Q(t) + \phi].$$
$$(2)$$

In Eq. (2), $A^{-1} = [-4\gamma^2 - 2\gamma^2\tanh(2t_0) - 2\cosh(2t_0)]$, $t_0 = \alpha L/2$, $t = \alpha(z - (L/2))$, $-t_0 \le t \le t_0$, $\tanh(\phi) = -\gamma(2/\cosh(2t_0))^{1/2}$, and $Q(t)$ is a phase function. E_2 and E_3 are obtained by plugging Eq. (2) into Eq. (1), and using the conservation law $|E_3|^2 = 1 - |E_1|^2 - |E_2|^2$. Figure 1(b) shows the evolution of the intensities $I_n = |E_n|^2$ in a 3-core adiabatic system with parameter values very close to those used in our experiments, as obtained from the analytical expressions of Eq. (2). The numerical results are not shown since they are very close to those already depicted. As clearly shown in Fig. 1(b), the power adiabatically leaves channel 3 and populates channel 1, with very little energy remaining in the intermediate waveguide, in perfect analogy to the STIRAP process. The first term on the right of Eq. (2) is primarily responsible for this adiabatic transition whereas the second one describes the oscillatory component in Fig. 1(b).

The waveguide triplet used in our experiment was fabricated on an AlGaAs substrate, using standard photolithography techniques [23]. The waveguides have a width of 3 μm and an effective index step of 0.0034. The sample length is $L = 18$ mm. The edge-to-edge distance between waveguides 1 and 2 is 2 μm at $z = 0$, and 7 μm at $z = L$, while the distance between waveguides 1 and 3 is fixed at 12 μm. This yields a coupling of 2500 m^{-1} between waveguides 2 and 3 at $z = L = 18$ mm and 250 m^{-1} between waveguides 2 and 3 at $z = 0$, while the coupling between the waveguides is 790 m^{-1} at $z = L/2$. A second sample with similar parameters was fabricated, and was truncated to enable observation of the amplitude in the waveguides before the full sweep is achieved. The light
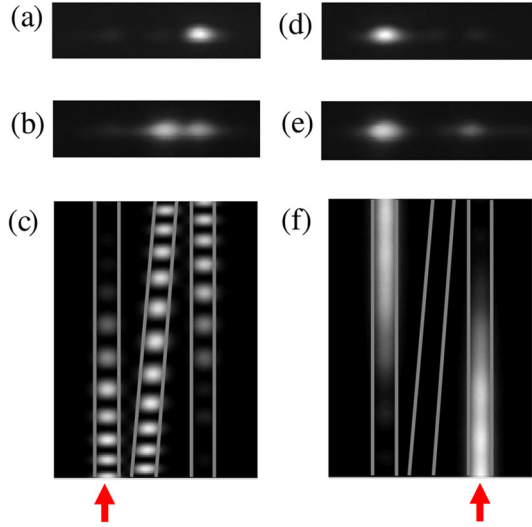
FIG. 2 (color online). Adiabatic passage in the STIRAP sample. (a) Measurement of the output light distribution when light is injected into waveguide 1. After the adiabatic sweep, the light emerges from waveguide 3. (b) The same experiment in a truncated sample, showing that during the adiabatic sweep, there is significant intensity in waveguide 2. (c) BPM simulations of the propagation. (d)–(f) The same as (a)–(c), when light is injected to waveguide 3 and emerges from waveguide 1. In this case, during the adiabatic sweep the intensity in waveguide 2 is negligibly small.

source is an optical parametric oscillator (Spectra-Physics OPAL) pumped by a mode-locked Ti:Sapphire laser, producing 130 fs pulses with 80 MHz repetition rate, at a wavelength of 1530 nm. In the experiments presented below, light is injected into one of the waveguides in the structure at $z = 0$, propagates along the sample, and is measured at the sample output. Nonlinearity is introduced by increasing the power of the input beam. The nonlinear parameter is evaluated to be $\Gamma \approx 5 \text{ m}^{-1}\text{W}^{-1}$ [24]. A full description of the setup can be found elsewhere [23,24].

Figure 2 shows the result of experiments done at low powers. When the input beam is launched into waveguide 1 [Fig. 2(a)], the output light emerges from waveguide 3. However, a similar experiment done in the truncated sample [Fig. 2(b)], reveals that waveguide 2 carries a significant field amplitude during the power exchange between waveguide 1 and waveguide 3. This is also illustrated in the beam propagation method (BPM) simulation shown in Fig. 2(c). On the other hand, when light is initially injected into waveguide 3, it emerges from waveguide 1 as shown in Fig. 2(d), yet the truncated sample shows that in this case the intensity in waveguide 2 is negligible during the process [Fig. 2(e)]. This result is corroborated by the BPM simulation, as shown in Fig. 2(f). Even though the coupling between waveguide 3 and waveguide 1 is zero, power is fully exchanged between them during the adiabatic sweep, with only minimal excitation of waveguide 2.

We now turn to the effect of nonlinear perturbations on the adiabatic passage described above. For this purpose we

again launched light into waveguide 3, and measured the output light distribution as a function of the input beam power. The results of this experiment are presented in Fig. 3(a). These results show that the presence of nonlinearity reduces the efficiency of the adiabatic passage, even at relatively weak powers (i.e., $\Gamma |E_n|^2 \simeq C_{n,m}$). The experimentally measured light distribution at the output is compared to BPM numerical results in Fig. 3(b), taking into account corrections due to dispersion and cross-phase modulation effects [25]. The experimental and numerical results show good agreement at mild powers, while at higher powers the experiment deviates from the theory, probably due to nonlinear absorption effects. Figure 3(c) shows an example of the calculated evolution of the intensities in waveguides 1 and 3 along the propagation in the nonlinear regime (power of 350 W). This figure should be compared with the linear dynamics in Fig. 1(b).

These results are compatible with previous theoretical predictions that considered the mean-field dynamics of a Bose-Einstein condensate in a time-dependent triple-well trap [11]. The authors have shown that the adiabatic passage should break down when the magnitude of the nonlinear parameter Γ exceeds that of the detuning between levels. In the optical analogue, detuning is introduced when the waveguides have different propagation parameters β. In the configuration used here all three waveguides are identical; hence, the adiabatic passage is expected to break down even for weak nonlinearity.
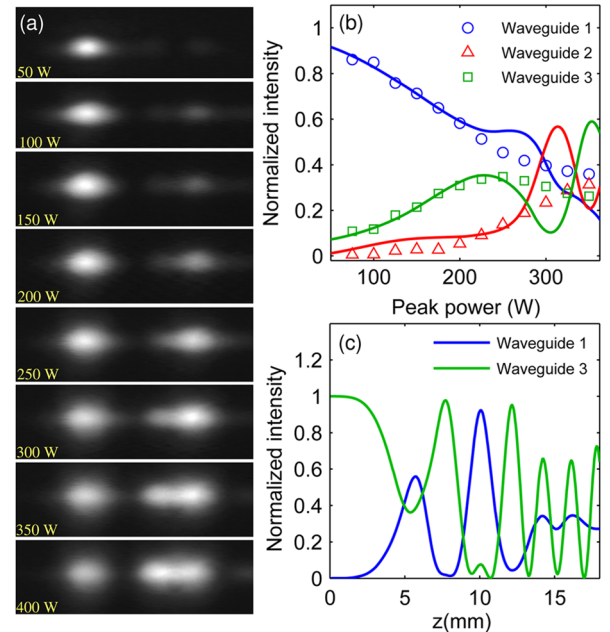

FIG. 3 (color online). The effect of nonlinearity on adiabatic passage. (a) Measurements of the output light distribution as in Fig. 2(d), at different input intensities. (b) comparison between the experimental results (markers) and numerical calculations (lines, see text). (c) Numerical calculations of the intensity distribution in the sample along the propagation, for an input power of 350 W.
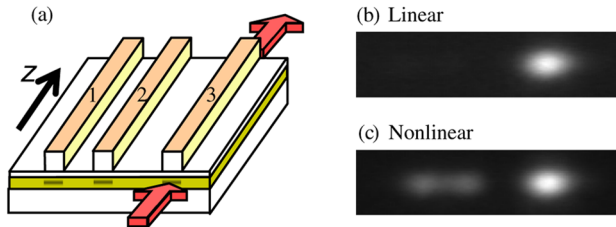
FIG. 4 (color online).   (a) A schematic view of the sample used to probe the dark state. (b) Formation of the dark state in the linear regime. Light is injected to waveguide 3 and remains trapped there, despite the coupling between waveguide 3 and waveguide 2 (see text). (c) Partial destruction of the dark state by nonlinearity.

The STIRAP effect relies on the existence of a dark eigenstate of the system, a phenomenon known as coherent population trapping (CPT) [6,9]. It has been theoretically shown that dynamical level shifts induced by nonlinearity can affect the resonance condition that leads to the CPT state, hence reducing the efficiency of STIRAP [12]. To demonstrate this effect in our system, we consider the configuration presented in Fig. 4(a) which is identical to the configuration of the STIRAP sample at $z = 0$, but with no variation of the couplings along the z direction. Waveguides 3 and 2 are weakly coupled; therefore, light injected into waveguide 3 is expected to tunnel along the propagation to waveguide 2. However, the strong coupling between waveguide 2 and waveguide 1 results in two new modes with propagation constants that are spaced symmetrically around that of the third guide. This leads to a sharp resonance that eliminates the tunneling, and therefore light that is injected into waveguide 3 remains trapped in that waveguide. The formation of this dark state is experimentally demonstrated in Fig. 4(b). When nonlinearity is introduced by increasing the input power (300 W), the eigenvalue of the mode in waveguide 3 is shifted and the resonance condition is no longer satisfied. As a result the dark state is destroyed and tunneling out of waveguide 3 is partially recovered [Fig. 4(c)]. Since the STIRAP effect is based on the evolution of this dark state, this also explains the sensitivity of STIRAP to nonlinearity. Is it interesting to note that even though the level detuning due to nonlinearity can in principle be compensated by the sample design, the dark state may still be dynamically unstable [12].

In summary, using coupled nonlinear optical waveguides we have investigated the effect of nonlinearity on an adiabatic process—an optical analogue of the STIRAP process. In the nonlinear regime, we found that nonlinearity impairs the efficiency of STIRAP. This was explained by the destruction of the dark state formed in the STIRAP configuration.

The approach presented here can be extended to more complex structures, implementing a variety of slowly varying potentials and giving rise to new nonlinear effects. Lattices can be used to adiabatically introduce changes in the dispersion relation, for example, by opening gaps in the spectrum or by introducing disorder, offering a new experimental playground for the study of the interplay between nonlinearity and adiabaticity.

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